Price Stability and Debt Sustainability under Endogenous Trend Growth

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Abstract

This paper studies price stability and debt sustainability when the real rate exceeds trend growth ($r > g$) in a New Keynesian model with endogenous technology growth through R&D. Under debt-stabilizing ("passive") fiscal policy the Taylor principle is not sufficient for determinacy. Instead, monetary policy should at least aim to raise $r - g$ with persistent inflation in order to stabilize the expectations of households, firms *and* innovators. Endogenous growth provides a self-financing mechanism for deficits under active fiscal policy; growth provides some backing for the public debt, which reduces the need for debt-stabilizing inflation when current fiscal deficits are not backed by future fiscal surpluses. Because growth creates some fiscal space, a monetary policy that adheres to the Taylor principle combined with active fiscal policy can yield a unique stable equilibrium, provided that the policy permits $r - g$ to fall with inflation.

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1. Introduction

The long-run trend is a central determinant of the debt-to-GDP ratio, which matters for price stability and fiscal sustainability considerations. However, standard results concerning monetary-fiscal interactions are grounded in models that abstract from the determinants of the long-term aggregate output path. These modeling paradigms support a particular view of the "appropriate" monetary-fiscal framework. That view associates price stability with a monetary policy that satisfies the "Taylor Principle" by raising nominal interest rates more than one-for-one with inflation. The success of such a monetary policy framework may depend on the exogeneity of economic factors affecting the long-run growth trend. Moreover, such a policy may put strain on government finances during times when inflation and the public debt are both high. As such, an anti-inflationary central bank tacitly relies on the fiscal authority to stabilize fiscal imbalances ("passive" fiscal policy). Without a passive fiscal surplus policy backing the public debt, monetary policy must violate the Taylor Principle and therefore let r fall with a persistent rise in inflation (Leeper (1991)). Drawing from the well-established result from the endogenous growth literature that technology growth through research and development (R&D) constitutes the main driver of long-run growth, this paper reevaluates fundamental questions concerning monetary-fiscal interactions in a New Keynesian model with endogenous long-run trend dynamics determined by the presence of entrepreneurs who invest in technological innovation through R&D.

We show that accounting for general equilibrium effects on the long-run trend margin alters the conditions for price stability and debt sustainability and the interaction between monetary and fiscal policy. Our insights stem from a central mechanism: changes in aggregate demand affect entrepreneurs' incentives to innovate, which generates endogenous adjustment in R&D investment, technology growth and an endogenous long-run path of aggregate output. Under an endogenous long-run growth trend, a monetary policy which satisfies the Taylor Principle is not sufficient for local determinacy when fiscal policy is passive. Instead, monetary policy should adhere to the *growth-augmented Taylor principle (GTP)* which formalizes a more stringent requirement with respect to the trend output growth rate (g): the central bank should raise $r - g$ in response to a persistent rise in inflation, and not just r as suggested by the Taylor Principle. The feedback between expected inflation and the payoff from technological innovation through R&D warrants this especially hawkish monetary policy stance. Consequently, the discrepancy between the Taylor Principle and GTP depends on the responsiveness of technology to aggregate demand.

On the other hand, violating the GTP is sufficient for stability under a debt-destabilizing ("active") fiscal policy. This result can be formulated in terms of a *dynamic* $r - g$ criterion: unbacked fiscal expansions can be financed through a combination of inflation and changes in the long-run output trend, so long as the monetary authority lets $r - g$ fall dynamically with a persistent rise in inflation. A monetary policy that violates the GTP rules out changes in the government's debt-service costs that offset the debt-stabilizing effect of inflation and technological innovation. The latter fiscal-financing margin is absent under exogenous growth (g is constant), and as a result, violation of the Taylor Principle is a strict requirement for fiscal sustainability in standard macro models. Because the GTP imposes a stricter requirement than the Taylor Principle, a model with endogenous growth can admit a unique stable equilibrium under active fiscal policy and a monetary policy which follows the Taylor principle. These findings highlight that demand-induced changes in trend growth can provide some backing for the public debt. This backing coming from trend growth dampens the changes in inflation needed to reduce public debt or finance new fiscal expenditures which are not backed by future fiscal revenues. In special cases in which technology is highly sensitive to demand, a fiscally-unbacked fiscal expansion can actually result in *deflation*. In all cases, endogenous growth relaxes the conditions for debt sustainability.

Our mechanism operates in a tractable representative agent model in which $r > g$ holds and which nests the standard three equation New Keynesian model. The model is populated by a representative household that consumes final output, holds bonds and is the owner of the firms. The household supplies two types of labor: unskilled labor, for goods production, and skilled labor, for technological innovation through R&D. There are two layers of production. A unit mass of final good firms produces differentiated final output goods under monopolistic competition using intermediate inputs. Their price setting is subject to a Calvo pricing friction. Intermediate goods are imperfect substitutes in final goods production and intermediate good firms produce using unskilled labor under monopolistic competition. The aggregate technology stock is a function of the number of intermediate good varieties which expands endogenously through R&D. Entrepreneurs in the R&D sector create new intermediate goods varieties using skilled labor and obtain the payoffs from a new intermediate good production line. The monetary authority sets nominal interest rates through a Taylor rule. The fiscal authority issues public debt and sets taxes via a fiscal surplus rule. These ingredients combine the monetary policy framework of Woodford (2003) with the endogenous growth model of Romer (1990), resulting in a simple framework ideally suited for the study of fundamental price stability and fiscal sustainability issues. We describe in what follows how this paper contributes to the previous literature.

Previous literature

Monetary-fiscal interaction and the FTPL. The fiscal theory of the price level (FTPL) literature, which follows Leeper (1991), Sims (1994), Woodford (1995), among others, emphasizes that the joint behavior of monetary and fiscal policy determines prices in a manner that ensures fiscal backing for the public debt.[1] The monetary authority stabilizes prices by conducting an "active" or anti-inflationary monetary policy, but only if the fiscal authority maintains a debt-stabilizing or "passive" fiscal stance at the same time. If instead, the fiscal authority behaves "actively" and without regard for fiscal sustainability, then price and debt stability requires a passive monetary stance. Macroeconomic stability therefore requires one active and one passive authority. This paper contributes to this literature by studying monetary-fiscal interaction under endogenous technology growth which, as Cochrane (2022) also observed, has not been previously analyzed in the FTPL literature. This paper is the first to study fiscal theory considerations with fully endogenous technology growth through R&D. We show that the typical assumption of an exogenous technology stock has non-trivial implications for monetary-fiscal interaction as

[1] See Cochrane (2022) and Leeper and Leith (2016) for comprehensive reviews of this literature.

it omits the long-run trend as an additional adjustment margin. Endogenous movements in technology growth require monetary policy to respond more hawkishly under passive fiscal policy, but endogenous growth also relaxes the conditions for debt sustainability under active fiscal policy compared with standard results. Crucially, an anti-inflationary monetary policy which follows the Taylor principle can be consistent with a unique stable equilibrium under active fiscal policy. Our model features a self-financing channel in which endogenous trend growth can create fiscal space and reduce the need for fiscal inflation.

Relation to $r < g$. A growing literature focuses on episodes in which the real interest rate ranges below the long-term growth rate of the economy, and derives models which can generate $r - g < 0$ (e.g., see Blanchard (2019), Mehrotra and Sergeyev (2021), Reis (2022)). As in this literature, we emphasize the importance of r and g for debt sustainability. However, our methodology, underlying mechanisms, and outcomes diverge significantly. First, we work with a representative agent model in which $r > g$, both dynamically and at the balanced growth path. Second, we link transitory changes in $r - g$ around its balanced growth path level to both price stability and fiscal sustainability. Further, we isolate assumptions about the joint conduct of monetary and fiscal policy that give rise to debt- and price-stabilizing dynamics of $r - g$ in equilibrium. Finally, our results highlight that growth does not unambiguously relax the conditions for macroeconomic stability. In fact, a more hawkish monetary policy stance is necessary for price stability under a passive fiscal policy. Similarly, monetary policy cannot be too hawkish for stability under active fiscal policy.

Conditions for Stability. A strand of the literature shows how conditions for macro stability depend on the features of the economy not included in standard models, such as the possibility of regime change (Ascari et al. (2020), Cho and Moreno (2021)), the presence of sovereign risk premia (Bonam and Lukkezen (2019)), bubble terms (Brunnermeier et al. (2022)), the presence of partially unfunded debt (Bianchi et al. (2023)), deviations from rational expectations (Eusepi and Preston (2012), Eusepi and Preston (2018)), or finite lives

(Angeletos et al. (2023)).[2] Our analysis complements this latter strand of papers by revealing how the endogeneity of technology and growth affects the conditions for stable prices and debt.

Endogenous growth. This paper models long-run trend dynamics as proposed by the endogenous growth literature, which identifies investment in R&D as the key driver of long-run growth (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). The mechanisms underlying this paper are well-documented empirically. Ma and Zimmermann (2023), Jordà et al. (2020) and Moran and Queralto (2018) demonstrate the long-term effects of monetary policy shocks on TFP through R&D, and Ilzetzki (2023) shows that exogenous aggregate demand increases raise technology growth.[3] Antolin-Diaz and Surico (2022) and Cloyne et al. (2022) show the long-run trend effects operating through R&D of government spending shocks and corporate tax cuts, respectively.

Our paper is closely related to the literature that introduces endogenous technology growth through investment in innovation in otherwise standard New Keynesian DSGE models. However, our paper is the first to introduce public debt and study monetary-fiscal interaction under endogenous growth. Our approach to incorporating endogenous growth via expanding varieties through R&D as in Romer (1990) into a simple New Keynesian environment is closely related to the approach taken by Queraltó (2022), who studies optimal monetary policy in a model without public debt. Garga and Singh (2021) study optimal monetary policy in a New Keynesian DSGE model with Schumpeterian growth through R&D. Estimated, medium-scale DSGE models with endogenous growth through innovation investment include Moran and Queralto (2018), Anzoategui et al. (2019), Bianchi et al. (2019), Ikeda and Kurozumi (2019), and Elfsbacka Schmöller and Spitzer (2021) and key insights include the effect of demand and monetary policy shocks

[2] In models without public debt, the Taylor Principle may not be sufficient for determinacy if capital and investment are present (Carlstrom and Fuerst (2005)), or under an ad hoc learning-by-doing mechanism (Micheli (2018)), or in a HANK model with counter-cyclical income risk (Acharya and Dogra (2020), Bilbiie (2021)). The Taylor Principle may not be necessary for determinacy if agents are myopic (Gabaix (2020)), have finite planning horizons (Woodford and Xie (2022)), social memory frictions (Angeletos and Lian (2023)), imperfect common knowledge (Angeletos and Lian (2018)), face a cost channel (Beaudry et al. (2024)), or in a HANK setting with pro-cyclical income risk (Acharya and Dogra (2020)).

[3] Further, Furlanetto et al. (2021) estimate the long-term effects of demand shocks and Elfsbacka-Schmöller et al. (2023) show the depressing effect of contractionary demand shocks on innovation investment at the firm level.

on innovation investment and TFP growth. Elfsbacka Schmöller (2022) studies multipliers of targeted fiscal stimulus to R&D and technology adoption. Benigno and Fornaro (2018) study self-fulfilling stagnation traps at the ZLB and Fornaro and Wolf (2020) study the long-run effects of supply shocks, such as a pandemic, in Keynesian growth models, i.e. growth models with Keynesian elements.

This paper is structured as follows. Section 2 presents the model framework. Section 3 revisits some basic results from canonical exogenous growth models. Section 4 then derives conditions for stability under endogenous growth, and proposes a generalization of the Taylor Principle. Section 5 provides reasoning for the result that growth relaxes conditions for debt sustainability. Section 6 discusses the importance of $r - g$ for stabilization policy. Section 7 concludes.

2. Model

We proceed to describe our theoretical framework, which is a tractable New Keynesian model with endogenous technology growth through R&D and public debt. As in Queraltó (2022), our model operates in the standard 3-equation New Keynesian DSGE model environment, combined with endogenous growth through expanding varieties as in Romer (1990), and novel to this paper, public debt.[4] A representative household consumes final consumption goods, holds bonds and owns the firms in the economy. The household supplies two types of labor: unskilled labor used in the production of intermediate goods and skilled labor which serves as input in the research and development sector. Production occurs in two layers of production. A unit mass of final good firms produces differentiated final consumption goods under monopolistic competition with intermediate goods as the only input. They face Calvo price rigidities and set prices in a staggered manner. Intermediate goods are imperfectly substitutable in final goods production. Intermediate good varieties are produced using unskilled labor as an input un-

[4] Queraltó (2022) studies optimal monetary policy under endogenous growth through R&D. Estimated medium-scale DSGE models typically also model growth through expanding varieties in intermediates following Romer (1990), but also model technology adoption, as in Comin and Gertler (2006). Due to our focus on analytical results we keep both the New Keynesian DSGE elements and the endogenous growth process tractable.

der monopolistic competition. Aggregate technology and thus TFP growth is driven by the number of intermediate good varieties which are endogenously determined through research and development. R&D entrepreneurs create new intermediate goods varieties with skilled labor as input and receive the payoffs from a newly created intermediate good production variety. Monetary policy sets nominal interest rates to target inflation and output via a Taylor rule. Fiscal policy issues public debt, makes fiscal expenditures, and sets taxes through a fiscal surplus rule.

Endogenous trend growth: The aggregate technology stock, A_t, is subject to endogenous growth. Technologies, i.e. intermediate good varieties, can become obsolete at rate $1-\phi$ and V_t denotes newly created technologies created through R&D in period t which become available for production in $t+1$. Thus, the technology stock, or total factor productivity, is governed by the process

$$A_{t+1} = \phi A_t + V_t \tag{1}$$

which states that the time $t+1$ technology stock equals to the surviving technologies from the previous period, ϕA_t, and technological innovations, V_t, created in time t. The long-run trend or total factor productivity equals to $A_t^{\frac{1}{\vartheta-1}}$, where ϑ is the elasticity of substitution in intermediates, and trend growth is a function of technology growth ($g_{y,t} = (g_{A,t})^{\frac{1}{\vartheta-1}}$ with $g_{A,t} = \frac{A_{t+1}}{A_t}$). We describe innovation and growth in detail in section 2.3.

2.1 Monetary policy

The central bank sets the nominal interest rate following a standard Taylor rule. In the baseline, the central bank targets inflation π_t and an output target as described by the interest rate rule

$$R_t = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}. \tag{2}$$

R_t denotes the risk-free gross nominal interest rate. As is conventionally the case, the central bank targets zero inflation ($\pi^* = 1$) and Y_t/Y_t^* denotes a standard output gap target defined later.

Definition. Monetary policy adheres to the **Taylor principle** if it raises interest rates more than one-for-one in response to a persistent increase in inflation. This holds true for ϕ_π, ϕ_y such that $\phi_\pi - 1 + \frac{(1-\beta)}{\kappa}\phi_y > 0$.

A monetary policy which follows the Taylor principle is typically described as "active" in the literature and passive otherwise.[5]

2.2 Fiscal policy

The fiscal authority faces the following intertemporal budget constraint:

$$B_t P_t^m + T_t P_t \quad = \quad B_{t-1}(1 + \rho P_t^m) + G_t P_t$$

where P_t denotes the price level and T_t real lump-sum taxes. B_t is the nominal bond portfolio which is subject to a a geometrically decaying coupon payment structure, i.e. one unit of the portfolio purchased at time-t at price P_t^m pays one unit of nominal income in $t+1$, ρ units in $t+2$, ρ^2 in $t+3$, and so on.

Expressed in terms of detrended real government debt[6]

$$b_t P_t^m + \tilde{T}_t \quad = \quad b_{t-1} \frac{(1 + \rho P_t^m)}{\pi_t g_{y,t-1}} + g_t \tag{3}$$

where $b_t = \frac{B_t}{P_t A_t^{\frac{1}{\vartheta-1}}}$, $\tilde{T}_t = \frac{T_t}{A_t^{\frac{1}{\vartheta-1}}}$, $g_{y,t} = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{\vartheta-1}}$.

We assume that government purchases are a fraction, $\tilde{g} > 0$, of total output in a bal-

[5]Section 3 revisits in detail the standard results as to the Taylor principle and its implications for active/passive monetary and fiscal policy.

[6]See Leeper et al. (2017) for estimated model expressed in terms of detrended real government debt.

anced growth path equilibrium. Government spending is characterized by the following process:

$$\frac{g_t}{\bar{g}} = \left(\frac{g_{t-1}}{\bar{g}}\right)^{\rho_g} \epsilon_t^G, \tag{4}$$

where $\bar{g} = \tilde{g}\bar{y} > 0$ denotes government spending on the balanced growth path, ϵ_t^G is i.i.d. mean one and $0 \leq \rho_g < 1$.[7] This implies the following market clearing condition

$$y_t = g_t + c_t, \tag{5}$$

where $y_t = Y_t/(A_t^{(\vartheta-1)^{-1}})$ and $c_t = C_t/(A_t^{(\vartheta-1)^{-1}})$.
The fiscal surplus rule is of the form:[8]

$$\frac{\tilde{T}_t}{\bar{\tilde{T}}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\frac{\gamma \bar{P}^m \bar{b}}{\bar{\tilde{T}}}} \tag{6}$$

Throughout the following analysis we are going to distinguish between active and passive fiscal policy according to the following definition:

Definition. Fiscal policy is said to be **passive** if $|\beta^{-1} - \gamma| < 1$. Otherwise, fiscal policy is said to be **active**.

As we discuss below, passive fiscal policy ensures fiscal adjustments that bring debt back to its long-run steady state level following any sequences of shocks, inflation, growth or interest rates. Active fiscal policy fails to guarantee debt-stabilizing adjustments in the fiscal surplus.

[7]We abstract from public R&D spending and our results may therefore understate the role of endogenous growth. In practice, public R&D spending may be increasing in government spending and public debt. Hence, public R&D would have direct effects on R&D, in addition to indirect effects on private R&D, which would reinforce our results.

[8]The log-linearized version of (6) implies that the log deviation of the real detrended fiscal surplus from the steady state balanced growth path is proportional to the log deviation of the real detrended public debt from the steady state balanced growth path. Note that we calibrate steady state $\tilde{T}$ ($\bar{\tilde{T}}$) to target a strictly positive debt-to-GDP ratio $dy > 0$: $\bar{b} = dy * \bar{y} > 0$.

2.3 Growth

Trend growth is modeled endogenously in general equilibrium. A continuum of measure one of innovators engage in research and development to generate new intermediate inputs. A successful innovator obtains the patent for the new innovation. Skilled labor serves as the R&D input. A newly created technology in t becomes available in production in period $t+1$. Each technology is subject to an exogenous probability of obsolescence ϕ ($0 < \phi < 1$). Innovation results in the generation of new technologies, i.e. an expansion of intermediate good inputs in the spirit of Romer (1990). As described in the beginning of section 2, the technology stock evolves as $A_{t+1} = \phi A_t + V_t$.

Research and development:

Innovators engage in research and development to generate new varieties of intermediate goods. Skilled labor, L_t^s, serves as R&D input and an innovator which creates a new intermediate good variety obtains the patent and thus the real profits, Π_t, from this respective intermediate production line. The payoff from a new technological innovation, J_t, equals to the expected present value of profits from the respective intermediate good production line

$$J_t = \mathbb{E}_t \left\{ \sum_{k=1}^{\infty} \phi^{k-1} \beta^k \frac{U_{C,t+k}}{U_{C,t}} \Pi_{t+k} \right\}. \tag{7}$$

The payoff from an innovation can thus be expressed in terms of the discounted value of period $t+1$ intermediate goods profits and the discounted expected continuation value $J_t = \mathbb{E}_t \left\{ \beta \frac{U_{C,t+1}}{U_{C,t}} \left(\Pi_{t+1} + \phi J_{t+1} \right) \right\}$. One unit of skilled labor allocated to R&D creates Φ_t new technologies. The R&D production function of an individual innovator j then takes the form

$$V_t(j) = \Phi_t L_t^s(j) \tag{8}$$

where $L_t^s(j)$ denotes skilled labor and $V_t(j)$ newly created technologies by innovator j respectively. The R&D production technology, Φ_t, is characterized by

$$\Phi_t = \varsigma A_t (L_t)^\eta \tag{9}$$

where ς denotes R&D efficiency. R&D production technology entails the aggregate time t technology stock, A_t, and thus the existing technological knowledge stock facilitates R&D and future innovation. The learning-by-doing (Romer (1986), Arrow (1962)) term, $(L_t)^\eta$, permits for complementarities between production and innovation for $\eta > 0$.[9]

Innovators' problem: Innovator j choose skilled labor, $L_t^s(j)$, to maximize the expected payoff from R&D (equ. 7) subject to the costs of R&D $\frac{W_{s,t}}{P_t} L_t^s(j)$. The optimality condition for R&D is given by

$$\Phi_t J_t = \frac{W_{s,t}}{P_t}. \tag{10}$$

Given symmetry, aggregate R&D input derives as $L_t^s(j) = L_t^s$ and $V_t = \int_0^1 V_t(j)$. Aggregate newly created technologies at time t obtain as

$$V_t = \Phi_t L_t^s. \tag{11}$$

From (11), the process governing the aggregate technology stock can be expressed as

$$A_{t+1} = \phi A_t + \Phi_t L_t^s. \tag{12}$$

Expanding varieties and TFP dynamics: As in estimated medium-scale DSGE models with endogenous growth through expanding varieties (Romer (1990)), R&D is the main driver of technology growth in this model framework. These quantitative frameworks[10] also feature costly adoption of new technology and a corresponding adoption choice, which slows down the diffusion of new technology to intermediates goods production. As the focus of this paper is on analytical results, we focus exclusively on R&D for tractability.[11] As shown by Comin and Gertler (2006), introducing lags in technological diffusion, increases the response of technology growth to short-run shocks, our

[9] See more detailed description at the end of this section.

[10] See Moran and Queralto (2018), Anzoategui et al. (2019), Ikeda and Kurozumi (2019), Elfsbacka Schmöller and Spitzer (2021), Cloyne et al. (2022) for estimated medium-scale DSGE models with growth through expanding varieties as in Romer (1990).

[11] Other papers which study endogenous technology growth in New Keynesian DGSE models with a focus on analytical results also focus on R&D as the main driver of technology growth (Queraltó (2022), Garga and Singh (2021)).

results should thus be interpreted as conservative in terms of the strengths of endogenous growth effects.

Complementarities between production and innovation: Ilzetzki (2023) provides empirical evidence that demand shocks[12] raise TFP more strongly under relatively tighter capacity as the latter spurs learning-by-doing (lbd) and innovation in response to the demand increase. Including lbd in the TFP process incorporates this mechanism, as above-trend production employment raises TFP growth and facilitates innovation. This property enhances realism in TFP dynamics in response to demand shocks by introducing complementary between production and innovation. We introduce lbd following Queraltó (2022) who calibrates η to match the TFP response following a monetary policy shock consistent with empirical estimates.

$\eta > 0$ introduces complementarities between production and innovation as higher production supports innovation. For $\eta = 0$ obtains the standard growth process for expanding varieties in intermediate goods through R&D as in Romer (1990). While lbd helps match the empirical TFP response to government spending and monetary policy shocks, we emphasize that lbd is not necessary for results, which obtain also for the case: $\eta = 0$.

2.4 Production and price setting

Intermediate goods firms operate in a monopolistically competitive environment using unskilled labor as input. Final good firms produce using intermediate goods as inputs and are subject to nominal pricing frictions.

2.4.1 Final goods producer

There is a continuum of measure one of monopolistically competitive final good firms which produce differentiated output $Y_t(i)$. The final good composite Y_t is the CES aggre-

[12] Ilzetzki (2023) studies demand increases following exogenous shifts in government spending.

gate of differentiated final good varieties:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{13}$$

Final good producers are subject to Calvo price rigidities. A final good firm produces according to the production function

$$Y_t(i) = \left[\int_0^{A_t} X_t(i,j)^{\frac{\vartheta-1}{\vartheta}} dj \right]^{\frac{\vartheta}{\vartheta-1}}. \tag{14}$$

$Y_t(i)$ denotes final output i and $X_t(i,j)$ the amount of intermediate good input j. $P_{x,t}(j)$ is the price of intermediate good variety j the demand for which can be derived as

$$X_t(i,j) = \left(\frac{P_{x,t}(j)}{P_{x,t}} \right)^{-\vartheta} Y_t(i).$$

where $P_{x,t}$ describes the price index of intermediates and follows

$$P_{x,t} = \left[\int_0^{A_t} P_{x,t}(j)^{1-\vartheta} dj \right]^{\frac{1}{1-\vartheta}}.$$

Real marginal costs, MC_t, of production are identical for all final goods firms and can be derived as

$$MC_t = \frac{P_{x,t}}{P_t}.$$

A firm which resets its price in t sets the optimal price P_t^* and the related optimality condition can be stated as:

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} U_{C,t+j} \beta^j \theta^j \left(\frac{P_t^*}{P_{t+j}} - \frac{\epsilon}{\epsilon-1} MC_{t+j} \right) Y_{t,t+j} \right\} = 0 \tag{15}$$

where $U_{C,t+j} = (C_{t+j})^{-1}$ and $Y_{t,t+j}$ is the demand for a goods variety in period $t+j$ assuming the price for the goods variety was last reset in period t. The final goods price index P_t obtains as

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{16}$$

2.4.2 Intermediate goods producer

Intermediate goods firms produce intermediate output $X_t(j)$ ($j \in [0, A_t]$) and are produced using unskilled labor L_t by the production technology

$$X_t(j) \quad = L_t(j), \tag{17}$$

where $X_t(j)$ denotes intermediate good output of variety j. Dividends in real terms in time t, $\Pi_t(j)$, follow from the maximization problem

$$\Pi_t(j) \quad = \max_{P_{x,t}(j), X_t(j)} \left\{ \left(\frac{P_{x,t}(j)}{P_t} - \frac{W_t}{P_t} \right) X_t(j) \right\} \tag{18}$$

subject to

$$X_t(j) = \left(\frac{P_{x,t}(j)}{P_{x,t}} \right)^{-\vartheta} \int_0^1 Y_t(i) di \tag{19}$$

which uses market clearing for intermediate good j ($X_t(j) = \int_0^1 X_t(i,j) di$). The conventional pricing condition follows: $P_{x,t}(j) = \frac{\vartheta}{\vartheta - 1} W_t$.
Using the pricing index for intermediates we derive real marginal costs as

$$MC_t = \frac{\vartheta}{\vartheta - 1} \frac{1}{A_t^{\frac{1}{\vartheta - 1}}} \frac{W_t}{P_t} \tag{20}$$

Firm profits from intermediate good production lines obtain as

$$\Pi_t = \frac{\nu_t}{\vartheta} \frac{MC_t}{A_t} Y_t \tag{21}$$

with price dispersion $\nu_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$.[13]

2.5 Households

Households maximize utility

$$E_t \sum_{k=0}^{\infty} \beta^s \left(log(C_{t+k}) - \frac{(L_{t+k})^{1+\varphi}}{1+\varphi} - \chi \frac{(L^s_{t+k})^{1+\varphi}}{1+\varphi} \right)$$

where C_t denotes an index of final good consumption, L_t denotes labor supplied to intermediate good producers and L^s_t denotes skilled labor supplied to the R&D sector. The household budget constraint can be stated as

$$P^m_t B_t + R^{-1}_t B^s_t + \int_0^1 P_t(i) C_t(i) di = W_t L_t + W_{s,t} L^s_t + B_{t-1}(1 + \rho P^m_t) + B^s_{t-1} - P_t T_t + P_t D_t$$

where B_t denote the nominal bond portfolio, R_t the risk-free gross nominal interest rate, $W_{u,t}$ and $W_{s,t}$ the wage from skilled and unskilled labor respectively, T_t real lump-sum taxes, D_t dividends, and B^s_t short-term debt which is in net zero supply. The bond portfolio has a geometrically decaying coupon payment structure.[14] From the household optimization problem the following optimality conditions obtain

$$1 = \beta \mathbb{E}_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} R_t \right\} \tag{22}$$

$$(L_t)^{\varphi} C_t = \frac{W_t}{P_t} \tag{23}$$

$$\chi (L^s_t)^{\varphi} C_t = \frac{W_{s,t}}{P_t} \tag{24}$$

$$P^m_t = E_t \left\{ R^{-1}_t \left(1 + \rho P^m_{t+1}\right) \right\} \tag{25}$$

Equation (25) is the no-arbitrage condition resulting from the first-order conditions for short-term bond holdings and holdings of the bond portfolio.

[13] Some of the optimality conditions are expressed in terms of final good output Y_t. We describe aggregation in more detail in section 2.6.

[14] Specifically, one unit of the portfolio purchased at time-t at price P^m_t pays one unit of nominal income in $t+1$, ρ units in $t+2$, ρ^2 in $t+3$, and so on.

2.6 Equilibrium and balanced growth path

In equilibrium, $\int_0^1 Y_t(i)di = A_t^{\frac{1}{\vartheta-1}} L_t$ and final output, Y_t, is characterized by $Y_t = \nu_t^{-1} \int_0^1 Y_t(i)di$. Endogenous trend growth, $g_{y,t}$, is determined by technology growth, $g_{A,t}$, where $g_{y,t} = (g_{A,t})^{\frac{1}{\vartheta-1}}$ and $g_{A,t} = \frac{A_{t+1}}{A_t}$. At the balanced growth path (BGP), the economy is subject to constant trend growth: $g_y = (g_A)^{\frac{1}{\vartheta-1}}$. The rate of technology growth, g_A, at the BGP is endogenous and can be derived as $g_A = \phi + \varsigma(L)^{\eta} L^s$.[15]

The balanced growth path is characterized in terms of stationary variables

$$\{L_t, L_t^s, R_t, \pi_t, P_t, P_t^*, P_t^m, MC_t, g_t^a, g_t^y\}$$

and trend-stationary variables: $\{y_t, c_t, g_t, \frac{W_t}{P_t}, \frac{W_t^s}{P_t}, \frac{B_t}{P_t}, T_t\}$ with trend $A_t^{\frac{1}{\vartheta-1}}$ and $\{J_t, \Pi_t\}$ with trend $A_t^{\frac{2-\vartheta}{\vartheta-1}}$. In an equilibrium, the endogenous variables described above must satisfy the equilibrium conditions (equations (C.1)-(C.19) in Appendix C), given initial conditions, b_{-1} and P_{-1}, and the government spending shock $\{\epsilon_t^G\}$. See Appendix C for the details about the model equilibrium and linearization.

Exogenous TFP case: The model described in section 2 nests the standard 3-equation New Keynesian model with exogenous technology which permits a direct comparison between our results and earlier results from this conventional benchmark framework. We restore the exogenous technology case through the assumption of perfect substitutability of intermediate goods ($\vartheta \to \infty$).[16]

[15] In practice, we calibrate g_A and g_y by choosing ς. See Appendix C for details.

[16] In this setting, an increase in the number of intermediate goods does not translate into changes in aggregate TFP and the long-run trend component. An alternative way of restoring the exogenous technology New Keynesian model is by imposing zero R&D input ($L_t^s = L^s = 0$), thus holding technology growth constant ($A_t = \bar{A}$), which can be obtained for $\chi \to \infty$.

2.7 Linearized System

We denote a log-linearized variable by $\hat{z}$ and define $\tilde{c} := 1 - \tilde{g}$, and $g_{A,t} := A_{t+1}/A_t$. We assume that the central bank targets detrended output ($Y_t^* = A_t^{\frac{1}{\vartheta-1}}\bar{y}$).[17] The system of equilibrium conditions can be log-linearized and compactly expressed as[18]

$$\hat{g}_{A,t} = \eta\omega\left(\hat{y}_t - \bar{\beta}E_t\hat{y}_{t+1}\right) + \bar{\delta}E_t\hat{y}_{t+1} + \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}E_t\hat{g}_{A,t+1} \tag{26}$$

$$\hat{g}_{y,t} = (\vartheta - 1)^{-1}\hat{g}_{A,t} \tag{27}$$

$$\hat{y}_t = E_t\hat{y}_{t+1} - \tilde{c}(\hat{i}_t - E_t\hat{\pi}_{t+1} - \hat{g}_{y,t}) + (1-\rho_g)\tilde{g}\hat{g}_t \tag{28}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{y}_t - \kappa\frac{\tilde{g}}{\varphi\tilde{c}+1}\hat{g}_t \tag{29}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t \tag{30}$$

$$\hat{b}_t = (\beta^{-1} - \gamma)\hat{b}_{t-1} - \beta^{-1}(\hat{\pi}_t + \hat{g}_{y,t-1}) + (\frac{\rho}{g_y} - 1)\hat{P}_t^m + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t \tag{31}$$

$$\hat{P}_t^m = -\hat{i}_t + \frac{\beta\rho}{g_y}E_t\hat{P}_{t+1}^m \tag{32}$$

where $\bar{\phi} := \phi/g_A$, $g_y = g_A^{(\vartheta-1)} \geq 1$, $g_A \geq 1$, $\bar{\beta} := \beta\bar{\phi}$, $\bar{\varphi} := \varphi/(1-\bar{\phi})$, $\omega := (1+\varphi)/(1+\bar{\varphi})$, $\bar{\delta} := (\varphi+1)(1-\bar{\beta})/(1+\bar{\varphi})$. The presence of endogenous growth, $\hat{g}_{y,t}$, in the Euler equation (28) and government budget constraint (31) distinguishes the model from a textbook New Keynesian model.

3. Stability and Exogenous Trend Growth

We now revisit some conventional wisdom about policy and macroeconomic stability. Generally, feasible macroeconomic policies are to fulfill two fundamental tasks: 1) determine the price level and 2) stabilize debt. A vast literature established that the joint behavior of monetary and fiscal authorities can insulate the economy from these two sources of instability. The hawkishness of the central bank, which is captured by the magnitude of ϕ_π and ϕ_y, is frequently associated with the first form of stability. In particular, it is widely

[17]This matches the typical assumption prevalent in New Keynesian DSGE models and in central banks in practice.

[18]Note thate $\kappa = \lambda(\varphi + \tilde{c}^{-1})$ where λ is the coefficient on marginal cost in the linearized Phillips curve.

maintained that ϕ_π and ϕ_y should be large enough so that interest rates rise by more than-for-one to a persistent increase in inflation (the "Taylor Principle"), which is enough to rule out extraneous beliefs-driven fluctuations ("sunspots"). In the exogenous growth model examined by Woodford (2001) (i.e. (26)-(32) with $\vartheta = \infty$), the Taylor Principle boils down to:

$$\phi_\pi - 1 + \frac{(1-\beta)}{\kappa}\phi_y = \frac{\partial \hat{r}}{\partial \hat{\pi}} > 0,$$

where $\partial \hat{r}/\partial \hat{\pi}$ represents the long-run response of the real interest rate, $\hat{r}$, to a permanent rise in inflation. That is:[19]

$$\frac{\partial \hat{r}}{\partial \hat{\pi}} := \lim_{k\to\infty} \frac{\partial(\hat{i}_{t+k} - \hat{\pi}_{t+k+1})}{\partial \hat{\pi}_t} = \lim_{k\to\infty} \frac{\partial\left(\phi_\pi \hat{\pi}_{t+k} - \hat{\pi}_{t+k+1} + \phi_y \hat{y}_{t+k}\right)}{\partial \hat{\pi}_t} = \phi_\pi - 1 + \frac{(1-\beta)}{\kappa}\phi_y.$$

A monetary policy that satisfies this principle may eliminate positive, self-fulfilling feedback between expected inflation, demand and income. Suppose, for example, that people believe inflation goes up by a percentage point in all future periods. Under the Taylor Principle, equilibrium inflation does not validate the exogenous change in expectations; the implied promise by the central bank to raise the real interest rate would reduce demand, and hence equilibrium inflation through the Phillips curve. Exogenous changes in expectations do not seed multiple dynamically stable equilibria. In fact, there can be only one bounded equilibrium, and in this bounded equilibrium, inflation, output, and consequently interest rates and bond prices are in steady state ($\hat{y}_t = \hat{\pi}_t = \hat{i}_t = \hat{P}^m_t = 0$) in the absence of shocks.[20]

However, a bounded equilibrium may not exist under the Taylor Principle. Suppose there are no shocks, and that the Taylor Principle is satisfied (such that $\hat{y}_t = \hat{\pi}_t = \hat{i}_t =$

[19] See also Chapter 4 of Galí (2015) for an equivalent interpretation of the Taylor Principle.

[20] The adaptive learning literature provides a different logic for the stabilizing effect of active monetary policy; in a sequence of *temporary* equilibria in which expectations are formed adaptively, the Taylor Principles raises the real interest rate when expected inflation is high, thus reducing actual inflation and hence the level of inflation expectations formed in the next period. In this environment, the Taylor Principle can rationalize how a rational expectations equilibrium emerges through a process of statistical learning. This logic does not apply to our rational expectations framework. Under rational expectations, the Taylor Principle entails off-equilibrium promises that render all but one solution of the model dynamically stable, as explained above.

$\hat{P}_t^m = 0$ describes the only potential stable equilibrium). Then the government's intertemporal budget constraint, (31), becomes:

$$\begin{aligned}\hat{b}_t &= (\beta^{-1} - \gamma)\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + (\frac{\rho}{g_y} - 1)\hat{P}_t^m \\ &= (\beta^{-1} - \gamma)\hat{b}_{t-1}.\end{aligned}$$

Clearly $|\hat{b}_t| \to \infty$ for $\hat{b}_{-1} \neq 0$ unless fiscal policy is *passive*. If debt explodes ($|\hat{b}_t| \to \infty$), then $\hat{y}_t = \hat{\pi}_t = \hat{i}_t = \hat{P}_t^m = 0$ cannot be an equilibrium due to a breakdown in Ricardian equivalence. For example, if $\hat{b}_{-1} > 0$ and fiscal policy is active, then households understand that their bond wealth will increase without bound, and without a commensurate increase in their tax burden. Consequently, agents perceive the initial public debt stock ($\hat{b}_{-1} > 0$) as net wealth, and this spurs demand and inflation in the initial period. Under the Taylor Principle, an increase in inflation begets higher real interest rates, which implies higher debt service costs and hence higher debt, which in turn implies higher demand and inflation. Therefore, adhering to the Taylor Principle under active fiscal policy may expose the economy to both hyperinflation and explosive debt. Violating the Taylor Principle breaks this explosive feedback loop between debt, inflation and real interest rates, by ensuring that real debt service costs fall when debt, and therefore inflation, rises. The fall in real interest rates helps brings public debt to steady state, which neutralizes wealth effects of high public debt. Some simple analytics using the government's intertemporal budget constraint, (31), shed light on the importance of violating the Taylor Principle under active fiscal policy. For simplicity, suppose a short maturity structure ($\rho = 0$) and exogenous fiscal surpluses ($\gamma = 0$). The budget constraint, (31), becomes:

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + \hat{i}_t + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t.$$

Solving the constraint forward, substituting for $\hat{r}_{t+j} = \hat{i}_{t+j} - \hat{\pi}_{t+j+1}$, and taking expectations yields:

$$\underbrace{\hat{b}_{t-1} - \hat{\pi}_t}_{\text{Real Value of Debt}} = \underbrace{-\sum_{j\geq 0} \beta^{j+1} \frac{\bar{g}}{\bar{P}^m \bar{b}} E_t \hat{g}_{t+j} - \sum_{j\geq 0} \beta^{j+1} E_t \hat{r}_{t+j}}_{\text{Discounted P.V. of Expected Fiscal Surpluses}} . \tag{33}$$

Equation (33) implies that the real value of government debt must equal the discounted present value of expected fiscal surpluses in every period. Thus, a fiscal expansion at time-t (an increase in $\{\hat{g}_{t+j}\}_{j\geq 0}$) which lowers the first term on the right-hand-side of (33) must be offset by an increase in current inflation ($\hat{\pi}_t$) or a fall in the path of expected path of future real interest rates ($\{\hat{r}_{t+j}\}_{j\geq 0}$). The Taylor Principle implies positive co-variation in $\hat{r}_t$ and $\hat{\pi}_t$, which is not debt-stabilizing. Violating the Taylor Principle permits negative debt-stabilizing co-movements in inflation and real interests that offset the effect of a fiscal expansion on (33).

The above intuition points us toward a simple policy prescription: we need one active authority and one passive authority. Two passive authorities allows for multiple equilibria. Two active authorities gives non-existence of dynamically stable equilibrium. This long-standing wisdom about the appropriate monetary-fiscal framework is summarized by Theorem 1.

Theorem 1 *Consider the exogenous growth model ($\vartheta = \infty$):*

i. Under passive fiscal policy, the Taylor Principle is a necessary and sufficient condition for local determinacy.

ii. Under active fiscal policy, violating the Taylor Principle is a necessary and sufficient condition for local determinacy.

4. Stability and Endogenous Trend Growth

According to Theorem 1, policymakers should facilitate the correct co-movements between real interest rates and inflation to manage stability risks. However, managing the co-movement of real interest rates and inflation is not sufficient when trend growth is endogenous. A comparison of the Euler equation under exogenous growth and the same equation under endogenous growth provides useful intuition. Consider first the standard exogenous growth Euler equation (with no shocks, for brevity):

$$\begin{aligned}\hat{y}_t &= E_t\hat{y}_{t+1} - \tilde{c}(\hat{\imath}_t - E_t\hat{\pi}_{t+1}) \\ &= E_t\hat{y}_{t+1} - \tilde{c} \times \hat{r}_t.\end{aligned}$$

It is apparent that for given expected future income, the Taylor Principle promises to contract demand in response to an expected permanent increase in inflation. The analogous equation under endogenous growth can be expressed as:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \tilde{c}(\hat{r}_t - \hat{g}_{y,t}).$$

The last equation suggests that the appropriate monetary policy response to inflation entails adjustments in $\hat{r}_t - \hat{g}_{y,t}$, and *not* just $\hat{r}_t$. Importantly, the trend output growth rate, $\hat{g}_{y,t}$, is endogenous and responds positively to a permanent increase in expected future inflation:

$$\begin{aligned}\frac{\partial \hat{g}_y}{\partial \hat{\pi}} &:= \lim_{k\to\infty}\frac{\partial \hat{g}_{y,t+k}}{\partial \hat{\pi}_t} = \lim_{k\to\infty}(\vartheta-1)^{-1}\frac{\partial \hat{g}_{A,t+k}}{\partial \hat{\pi}_t} \\ &= \lim_{k\to\infty}(\vartheta-1)^{-1}\left(\eta\omega\frac{\partial \hat{y}_{t+k}}{\partial \hat{\pi}_t} + (\bar{\delta}-\bar{\beta}\eta\omega)\frac{\partial \hat{y}_{t+k+1}}{\partial \hat{\pi}_t} + \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\frac{\partial \hat{g}_{A,t+k+1}}{\partial \hat{\pi}_t}\right) \\ &= \frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1-\frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)}\lim_{k\to\infty}\frac{\partial \hat{y}_{t+k}}{\partial \hat{\pi}_t} \\ &= \frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1-\frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)}\left(\frac{1-\beta}{\kappa}\right) > 0\end{aligned}$$

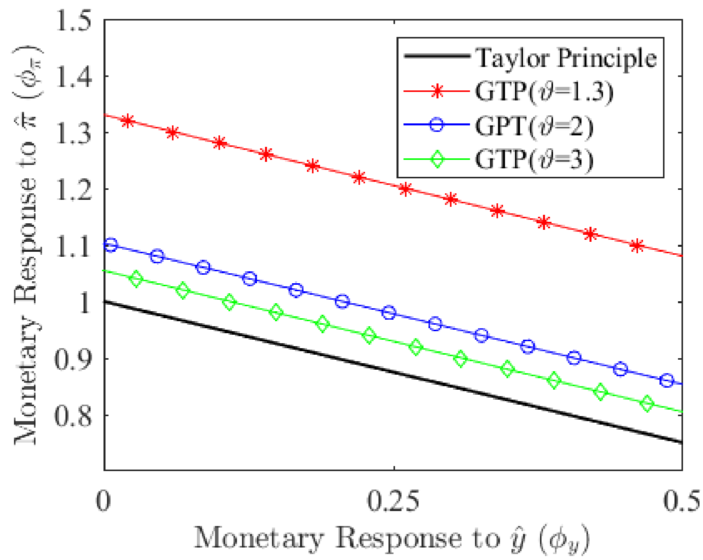
The fact that expected inflation raises trend growth reflects incentives to innovate; higher expected inflation implies higher aggregate demand, which raises the demand for input varieties and therefore the return to innovation and R&D. Thus, demand raises trend growth, and in turn, higher trend growth feeds back to aggregate demand through the Euler equation. Innovation through R&D therefore strengthens the overall responsiveness of demand to expected inflation, relative to the textbook exogenous growth case. Raising $\hat{r}-\hat{g}_y$ to lean against the inflation requires an especially hawkish monetary policy:

Definition. The **growth-augmented Taylor Principle (GTP)** is satisfied if and only if:

$$\phi_\pi - 1 + \frac{(1-\beta)}{\kappa}\left(\phi_y - \frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1-\frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)}\right) = \frac{\partial(\hat{r}-\hat{g}_y)}{\partial\hat{\pi}} > 0.$$

The GTP generalizes the Taylor Principle to the endogenous growth economy. It constitutes a stricter requirement for monetary policy, as any interest rate rule that satisfies the GTP will also satisfy the Taylor Principle, but not vice versa. The wedge between the GTP and Taylor Principle is given by $\partial\hat{g}_y/\partial\hat{\pi}$, which depends on the elasticity of substitution between intermediate varieties, ϑ, among other features of the economy. When intermediate varieties are highly imperfect substitutes in the production of final consumer goods (low ϑ), producers of intermediates have more market power and charge larger markups, which induces entrepreneurs to innovate relatively aggressively in response to a rise in demand. The resulting response of trend growth to a persistent rise in inflation, $\partial\hat{g}_y/\partial\hat{\pi}$, is larger in magnitude for lower values of ϑ, as depicted in Figure 1. As ϑ rises, the wedge between the traditional Taylor Principle and its growth-augmented counterpart vanishes. Simply put, our approach gives rise to a continuum of possible endogenous growth models, indexed by $\vartheta \in (1,\infty)$, and the growth-augmented Taylor Principle captures just how far we deviate from the conventional wisdom on macroeconomic stability when we choose a finite ϑ, consistent with the large and growing body of evidence in support of endogenous growth. For small departures from the exogenous growth case (very large ϑ), the growth-augmented and traditional Taylor Principle nearly coincide and the conventional wisdom applies for all practical intents and purposes. On the other hand, smaller ϑ delivers non-trivial and possibly large changes in the recipe for the right policy mix. Whether ϑ is large or small is an empirical question that depends on the country and time period under study, and which we do not answer in this paper. Following suggestive evidence from Broda and Weinstein (2006), a number of papers have calibrated $\vartheta \in [2, 4]$,[21] but Broda and Weinstein (2006) also give suggestive evidence that ϑ could be as low as 1.3.

Figure 1: Growth-augmented Taylor Principle



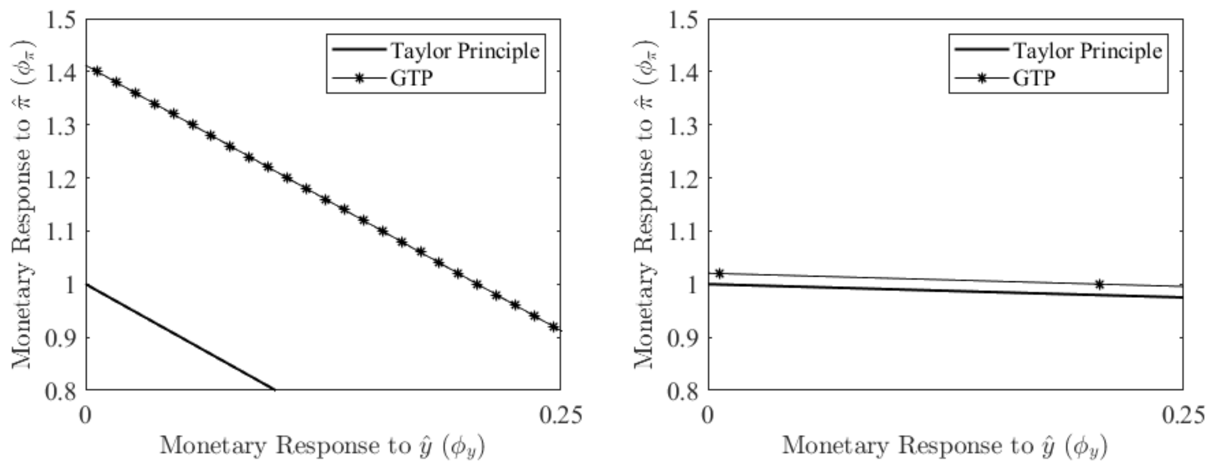
Notes: we assume $\beta = 0.99$, $\varphi = 2$, $\eta = 1.5$, $\kappa = 0.02$, $\phi = 0.925$, $g_y = (1.02)^{1/4}$, $\bar{c}/\bar{y} = 0.8$.

Additionally, the price rigidity faced by finals goods firms (captured by the slope of the Phillips curve, κ) shapes the wedge between the traditional Taylor Principle and its growth-augmented counterpart. In general, a flatter Phillips curve dampens the indirect pass-through from expected income to actual income through current and expected inflation in the Euler equation. Thus, for given $\phi_y > 0$, lower κ has a stabilizing effect on the economy even if it at the same time indirectly mitigates the effect that a high inflation reaction coefficient has on demand. The same is true in the presence of endogenous growth. The slope of the Phillips curve, however, has no mediating impact on the indirect response of actual income to expected income through trend growth. Lowering the slope of the Phillips curve only increases the feedback between expected and actual income through trend growth by reducing the effect that given ϕ_π has on aggregate demand. Consequently, we might be more concerned about the robustness of the conven-

[21]E.g., see Comin and Gertler (2006), Anzoategui et al. (2019), Elfsbacka Schmöller and Spitzer (2021), Queraltó (2022), among many others.

tional active-passive policy prescription when the Phillips curve is very flat, as depicted in Figure 2b.

Figure 2: Growth-augmented Taylor Principle and Price Rigidity



(a) $\kappa = 0.005$ (b) $\kappa = 0.1$

Notes: all other parameters calibrated as in Figure 1.

In a nutshell, the GTP modifies the Taylor Principle to reflect the possibility that innovation depends on expectations, and the extent to which innovation responds to those expectations depends on various structural features of the economy such as the elasticity of substitution between intermediate varieties and degree of price rigidity. However, whether a central bank should strive to satisfy the GTP unsurprisingly depends on whether fiscal policy is passive or active. Under an active fiscal policy, the central bank has reason to violate the GTP.

Proposition 1 *Consider the endogenous TFP model ($\vartheta < \infty$), and suppose η is sufficiently small as defined in Appendix B. Then*

i. Under passive fiscal policy, the growth-augmented Taylor Principle is a necessary condition for local determinacy.

ii. Under active fiscal policy, violating the growth-augmented Taylor Principle is a sufficient condition for local determinacy.

Proposition 1 is the main result of the paper. It adapts a conventional wisdom about price stability and fiscal sustainability to the possibility of endogenous trend growth through technological innovation. The proposition reveals some novel implications of growth for macroeconomic stability. First, growth necessitates an especially active monetary policy, for the reasons described above. Second, the GTP is not sufficient for price stability under passive fiscal policy, and in particular, a weak response of interest rates to the output gap can lead to indeterminacy, as depicted in Figure 3. Third, active fiscal policy imposes weak restrictions on monetary policy under endogenous growth relative to the exogenous growth case. In fact, a unique bounded equilibrium can exist when there is both active fiscal policy and "active" monetary policy in the sense that the Taylor Principle is satisfied, as made explicit by Corollary 1. However, endogenous growth does not imply a free fiscal lunch, and consequently monetary policy cannot be too anti-inflationary (i.e., a policy that satisfies the GTP can preclude existence of stable equilibrium under active fiscal policy).

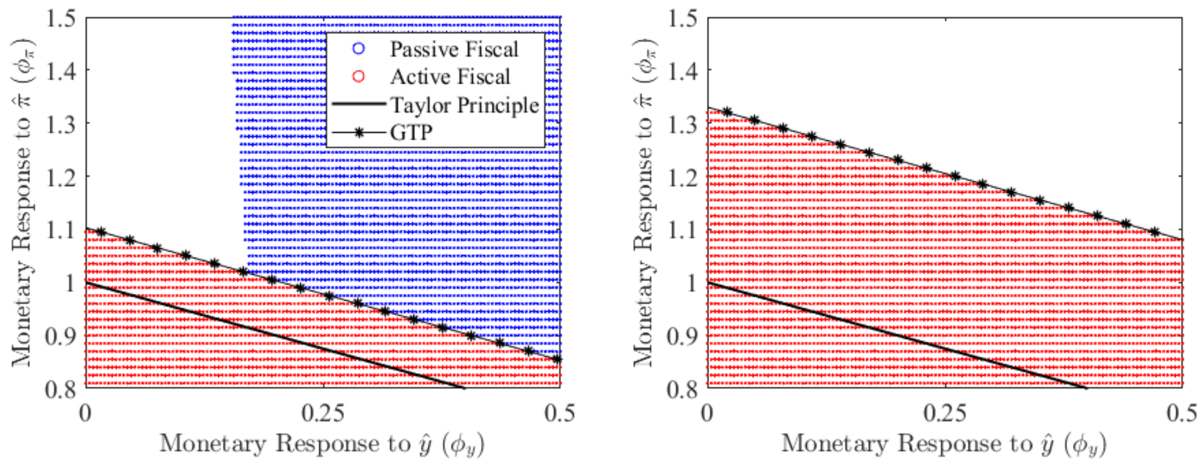
Corollary 1 *A unique bounded equilibrium exists under active fiscal policy and the Taylor Principle if the growth-augmented Taylor Principle is violated.*

The following section examines the interplay between growth and fiscal sustainability, and the corresponding conditions for stability under active fiscal.

5. Growth, Inflation, and Active Fiscal Policy

We next study the consequences of growth for inflation under active fiscal policy. Section 5.1 provides intuition for the key theoretical result that growth can generate fiscal space and alleviate the burden of fiscal inflation. We show that the maturity structure of debt and the strength of the endogenous growth mechanism are essential for the inflation response to high public debt and fiscal expenditures (sections 5.2 and 5.3, respectively).

Figure 3: Uniqueness and Existence



(a) $\vartheta = 2$ (b) $\vartheta = 1.3$

Notes: the blue (red) region is determinacy under passive (active) fiscal policy with endogenous growth. The white area corresponds to indeterminacy under active or passive fiscal policy. All other parameters calibrated as in Figure 1.

5.1 Growth creates fiscal space, reducing the need for fiscal inflation

Why does innovation through R&D relax the conditions for fiscal sustainability? We show that growth endogenously finances part of the public debt in an equilibrium with active fiscal policy, which reduces the need for debt-stabilizing inflation. In this sense, endogenous growth substitutes for the kind of fiscal inflation that is necessary for fiscal sustainability under active fiscal policy and exogenous growth, and which the central bank tolerates by violating the Taylor Principle. We attempt to establish the idea that endogenous growth substitutes for fiscal inflation in this section. We also document properties of an equilibrium with endogenous trend growth and active fiscal policy along the way.

To begin, consider the government budget constraint (31), and assume short maturity structure ($\rho = 0$) and exogenous fiscal surpluses ($\gamma = 0$) for simplicity:

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}(\hat{\pi}_t + \hat{g}_{y,t-1}) + \hat{i}_t + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t.$$

Unlike in the exogenous growth case ($\vartheta = \infty$), fluctuations in trend growth affect the evolution of debt. Higher growth yesterday implies lower debt today, all else constant. Solving the constraint forward, substituting for $\hat{r}_{t+j} = \hat{i}_{t+j} - \hat{\pi}_{t+j+1}$, and taking expectations yields:

$$\hat{b}_{t-1} - \hat{\pi}_t - \hat{g}_{y,t-1} = -\sum_{j\geq 0} \beta^{j+1} \frac{\bar{g}}{\bar{P}^m \bar{b}} E_t \hat{g}_{t+j} - \sum_{j\geq 0} \beta^{j+1} E_t \left(\hat{r}_{t+j} - \hat{g}_{y,t+j} \right). \quad (34)$$

Equation (34) is the endogenous growth analog of (33) from section 3. As in the exogenous growth case considered in section 3, an unbacked fiscal expansion (increase in $\{\hat{g}_{t+j}\}_{j\geq 0}$) which comes at time-$t$ lowers the discounted present value of expected fiscal surpluses (right-hand-side of (34)). To satisfy (34), inflation ($\hat{\pi}_t$) can reduce the real value of debt, or $\{\hat{r}_{t+j}\}_{j\geq 0}$ and $\{\hat{g}_{y,t+j}\}_{j\geq 0}$ can offset the effect of fiscal policy on the discounted present value term. Notice that the expected path of growth rates ($\{g_{y,t+j}\}_{j\geq 0}$) can also adjust to satisfy (33), unlike in the exogenous growth case considered in section 3. Thus, changes in current and expected future trend output growth can substitute for inflation or variation in the real interest rate and provide backing for the public debt. Moreover, the second sum on the right-hand-side of (34) reveals why violations of the GTP permit negative debt-stabilizing co-movements in inflation and $\hat{r} - \hat{g}_y$. Adhering to the GTP, on the other hand, means that the discounted present value of expected fiscal surpluses falls in inflation, which suggests that inflation will not finance a rise in fiscal expenditures.

Equation (34) gives some partial equilibrium intuition about the interplay between growth, inflation, and active fiscal policy when the GTP is violated. Those predictions are confirmed in general equilibrium: growth substitutes for fiscal inflation, and hence fiscal expansions permanently increase output and generate less fiscal inflation than predicted in canonical models of exogenous growth.

Showing that growth can substitute for inflation requires solving the model. Assuming that the GTP is violated and fiscal policy is active, a unique bounded equilibrium exists. The law of motion for inflation in the unique equilibrium assumes the form:[22]

$$\hat{\pi}_t = \Omega_{\pi,b} \hat{b}_{t-1} + \Omega_{\pi,g} \hat{g}_{A,t-1} + \Gamma_\pi \hat{g}_t \quad (35)$$

[22]The solution approach is detailed in Appendix A.

where, generically, the coefficients in (35) depend on the model parameters, and $\Omega_{\pi,b} = \partial\hat{\pi}_t/\partial\hat{b}_{t-1}$ and $\Gamma_\pi = \partial\hat{\pi}_t/\partial\hat{g}_t$ capture how inflation responds to high public debt and changes in fiscal expenditures, respectively. The special case of an interest rate peg ($\phi_\pi = \phi_y = 0$), active fiscal policy ($\gamma = 0$), no learning-by-doing ($\eta = 0$), and linear disutility in labor ($\varphi = 0$) allows us to cleanly examine how endogenous growth alters the general equilibrium response of inflation to fiscal variables.

Proposition 2 *Consider (26)-(32) and suppose $\varphi = \eta = \gamma = \phi_\pi = \phi_y = 0$. If growth is exogenous, then $\frac{\partial\hat{\pi}_t}{\partial\hat{g}_t} = \Gamma_\pi > 0$ and $\frac{\partial\hat{\pi}_t}{\partial\hat{b}_{t-1}} = \Omega_{\pi,b} > 0$.*

Proposition 2 confirms that inflation always rises during a fiscal expansion, or when public debt is high, in an exogenous growth economy. This "fiscal inflation" reflects a breakdown in Ricardian equivalence due to which agents perceive an increase in public debt as an increase in their net wealth. Without the inflation, public debt does not return to steady state following a shock to the government's finances. Under endogenous growth, the same coefficients in (35) are given by

$$\Omega_{\pi,b} = \frac{\kappa(1-\beta\lambda_1)}{\kappa+\beta\lambda_1(1-\beta\lambda_1)\mu} > 0, \tag{36}$$

$$\Gamma_\pi = \xi\Omega_{\pi,b}, \tag{37}$$

$$\xi = \left(\frac{\bar{g}(g_y-\beta\rho)}{\bar{b}(1-\beta\rho_g)} - \frac{2\beta\mu\rho_g\tilde{g}}{1+\beta-2\beta\rho_g+\beta(1+\tilde{c}\mu)\sqrt{\gamma_1^2-4\gamma_0}+\tilde{c}(\kappa+\mu(1-2\beta\rho_g))}\right),$$

where $\mu := (1-\bar{\beta})/(\vartheta-1)$, $\gamma_0 := \frac{1}{\beta(1+\tilde{c}\mu)}$, $\gamma_1 := \frac{1+\beta+\kappa\tilde{c}+\tilde{c}\mu}{\beta(1+\tilde{c}\mu)}$, $\lambda_1 := 0.5(\gamma_1 - \sqrt{\gamma_1^2-4\gamma_0}) \in [0,1)$.

A careful inspection of the last equations reveals two properties of the equilibrium with growth. First, we might expect inflation to respond very little to changes in public debt when the elasticity of substitution between input varieties is very low (i.e., μ is very high). Specifically, (36) suggests that $\partial\hat{\pi}_t/\partial\hat{b}_{t-1} = \Omega_{\pi,b}$ may be close to zero when ϑ is very small (section 5.2 explores this in more detail). Second, from (37), $\frac{\partial\hat{\pi}_t}{\partial\hat{g}_t} = \Gamma_\pi < 0$ if and only if $\xi < 0$. Therefore, endogenous growth opens up the possibility that inflation *falls* in fiscal expenditures. We analyze this case in detail in section 5.3.

5.2 Eroding the public debt

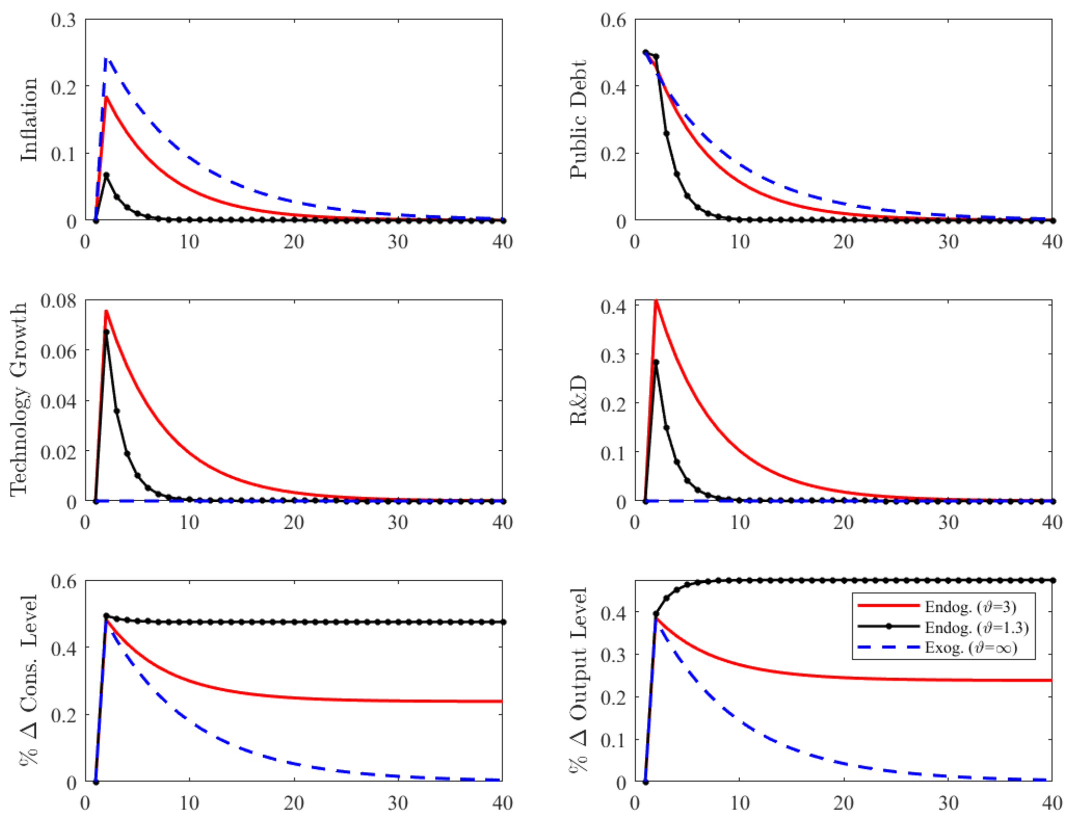
We study next debt and inflation dynamics starting from a situation in which the public debt level is elevated above the sustainable long-run level ($\hat{b}_{t-1} > 0$ in the context of our model at time-t).[23] Elevated debt levels, the risk they pose for inflation, and the question of how to revert debt to sustainable levels, have recently been identified as key challenges both by policymakers and the public debate, most notably following the COVID-19 crisis. As shown in Proposition 2, high debt levels raise the risk of fiscal inflation if the debt is not fiscally backed and trend growth is exogenous. We show that endogenous growth can reduce the inflationary effect of legacy debt, as indicated by equation (36). Figures 4 and 5 and Figure 6 panel (a) illustrate the inflation and growth effects of high public debt under more general assumptions (e.g., $\varphi > 0$, $\eta > 0$).

Those figures depict the impulse responses to an initial debt stock that is 50 percent above steady state level. Fiscal policy is assumed to be active ($\gamma = 0$) and the interest rate is pegged at steady state. Under exogenous growth, the debt is financed primarily via inflation, and without any adjustment in trend output growth. Under endogenous growth, the effect of demand on R&D and long-run growth help to bring debt back to steady state without as much inflation. The impulse responses capture the usual breakdown in Ricardian equivalence: unbacked high public debt raises aggregate demand, which leads to inflation under exogenous growth, but a mixture of demand-pull inflation and *disinflationary* technological innovation under endogenous growth.

Figures 4 and 5 show that the cumulative inflation generated by the high legacy debt depends on the strength of the endogenous growth channel, as captured by the magnitude of ϑ, and also on the steady state trend output growth rate. In particular, a country with relatively intense spillovers from demand to growth and/or higher trend growth rates may experience less inflation for a given deviation of debt from its steady state level. Countries such as the US, Japan and Italy are among the many economies experiencing elevated public debt levels, but there are considerable differences in trend output growth rates, the percentage of GDP devoted to R&D, and potentially the responsiveness of tech-

[23] The sustainable, long-run steady state debt level is determined by a host of factors which we do not address explicitly in our calibration of the debt-to-GDP ratio.

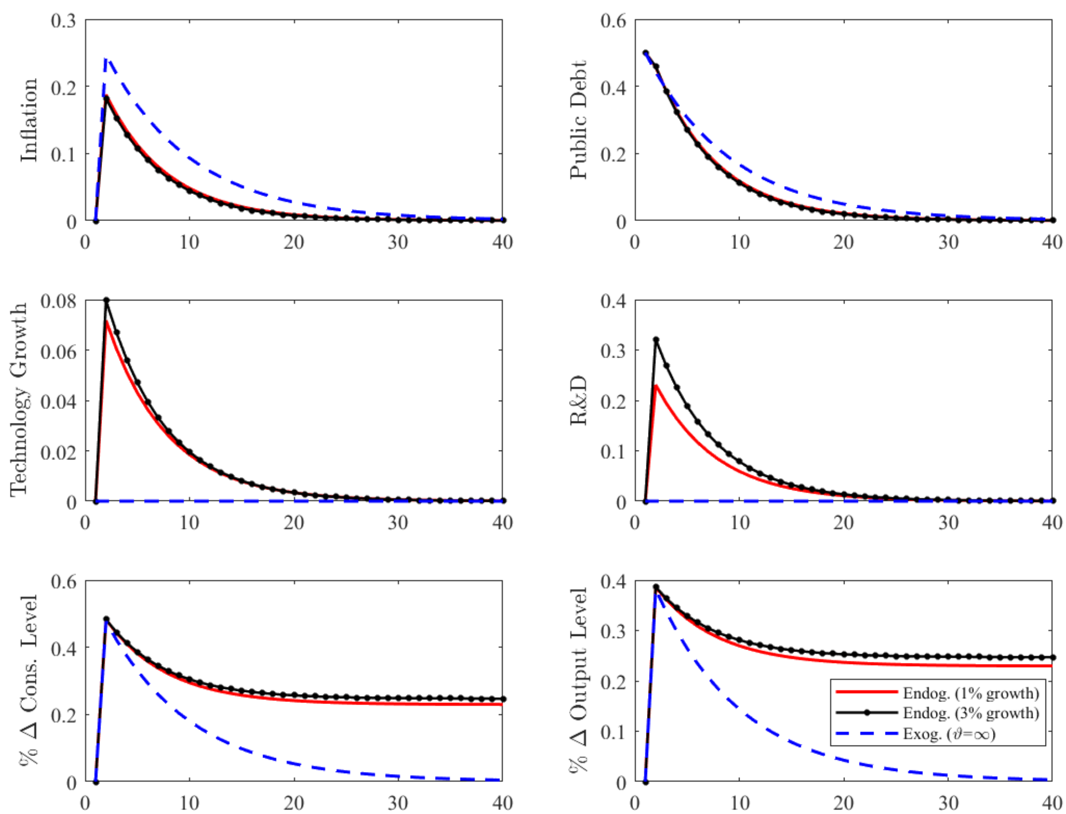
Figure 4: Eroding Public Debt: Role of ϑ



Notes: all other parameters calibrated as in Figure 1.

nological innovation and the trend to fluctuations in demand, across these countries. Our model suggests that quantitative predictions about the effects of this debt in different national economies should account for heterogeneity in the intensity of R&D and its role in TFP growth, as well as the balanced growth path trend output growth rate.

Figure 5: Eroding Public Debt: Role of g_y



Notes: $\vartheta = 3$ in reported simulations, and all other parameters calibrated as in Figure 1.

5.3 Duration of public debt: go long to keep inflation low

As stated in section 5.1, (36) suggests that $\partial\hat{\pi}_t/\partial\hat{b}_{t-1} = \Omega_{\pi,b}$ can be close to zero for very low ϑ. Moreover, (37), $\frac{\partial\hat{\pi}_t}{\partial\hat{g}_t} = \Gamma_\pi < 0$ if and only if $\xi < 0$. Notice that ξ is strictly decreasing in the duration of debt, ρ. Therefore, there exists a ρ^* such that $\partial\hat{\pi}_t/\partial\hat{g}_t = \Gamma_\pi \leq 0$ if and only if $\rho \geq \rho^*$.[24]

Proposition 3 *Consider (26)-(32) and suppose $\varphi = \eta = \gamma = \phi_\pi = \phi_y = 0$. Then $\frac{\partial\hat{\pi}_t}{\partial\hat{g}_t} = \Gamma_\pi$ is strictly decreasing in ρ, and $\frac{\partial\hat{\pi}_t}{\partial\hat{g}_t} = \Gamma_\pi \leq 0$ if and only if $\rho \geq \rho^*$.*

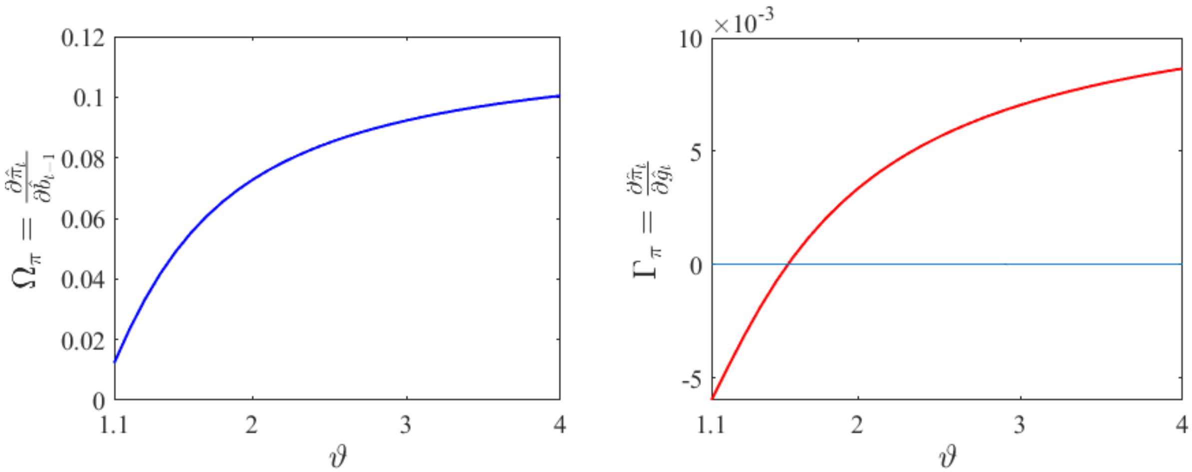
Intuitively, a longer debt maturity structure (higher ρ) delays the impact of a fiscal expansion on the government's finances, since lengthening the maturity structure reduces the share of debt that needs to be rolled over in a given period. Formally, higher ρ implies a smaller coefficient on $\hat{g}_t$ in the budget constraint (31) (because longer maturity raises the steady state bond price, $\bar{P}^m$). Since the financing needs of the government greatly impacts the amount of inflation generated by a fiscal expansion, we should expect lower inflation for longer maturity structures in the period the government raises its expenditures. At the same time, the direct relationship between aggregate demand, (28), innovation (26) and government spending is unaffected by the public debt maturity structure. Therefore, increasing ρ reduces the direct effect of fiscal policy on public debt, but does not change the direct effect of fiscal policy on aggregate demand. The former effect tends to be inflationary, while the latter effect is deflationary under endogenous growth.[25] The net effect depends on debt maturity structure.

Figure 6 shows that growth depresses the response of inflation to debt and fiscal expenditures under more general assumptions about η and φ than those considered above. When growth effects are strong (ϑ is small) inflation may fall in government spending, and changes in public debt have very little effect on inflation. The response of inflation to public debt and government spending is monotonically increasing in ϑ, which suggests that stronger growth effects dampen inflation responses to fiscal policy. We chose $\rho = 0.96$ for the calibration in order to roughly match the duration of public debt in the US. From the figure, it is apparent that this $\rho = 0.96$ is only greater than the implied ρ^* if ϑ is fairly small.

[24] We note that nothing in the model implies that $\rho^* \in [0, 1]$. If $\rho^* < 0$ ($\rho^* > 1$) then any maturity structure is consistent with deflationary (inflationary) fiscal policy.

[25] E.g., see D'Alessandro et al. (2019), Engler and Tervala (2018), or Jørgensen and Ravn (2022) concerning the deflationary effects of government spending in environments with endogenous growth.

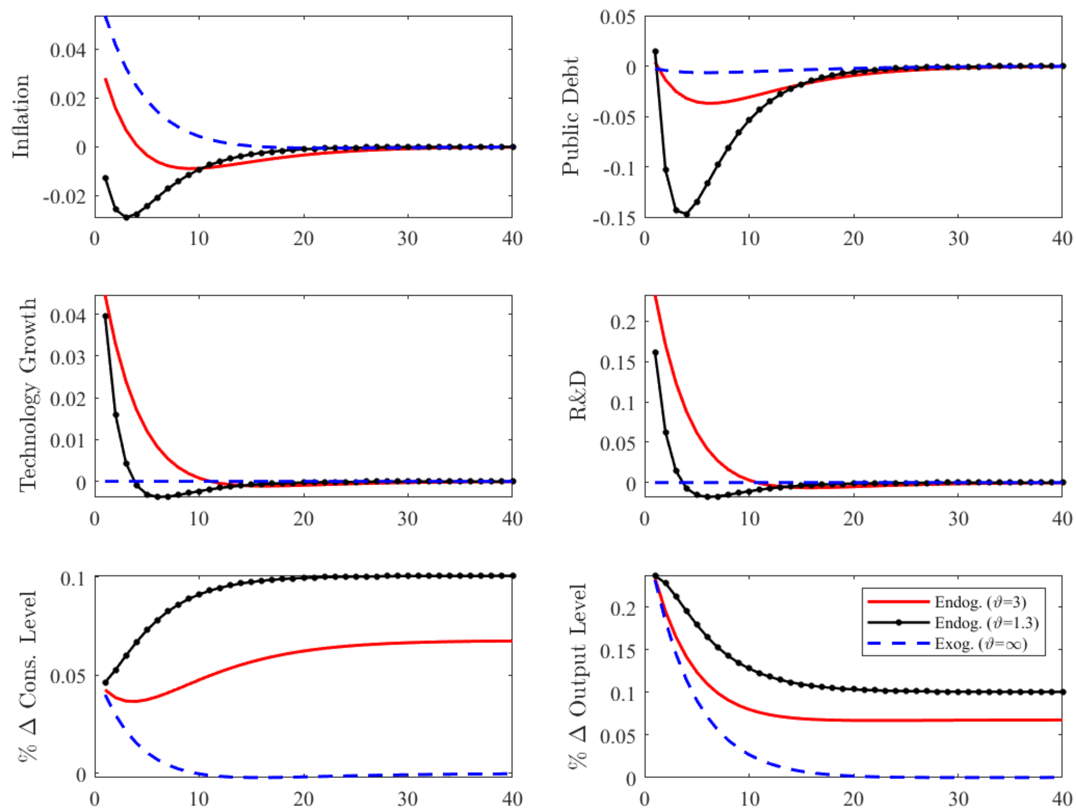
Figure 6: Inflation and Fiscal Policy



Notes: All other parameters calibrated as in Figure 1.

Our discussion of $\Omega_{\pi,b}$ and Γ_π concerns only the effect of fiscal policy on impact. Figure 7 instead depicts impulse responses to a persistent increase in government spending under different assumptions about the strength of the spillover (ϑ). Following an expansionary shock to government spending under active fiscal policy, the inflation response is attenuated in the models with endogenous technology (red and black lines), compared with the exogenous technology model (blue line). This is the case as the initial increase in public debt, perceived as an increase in net wealth by households, raises demand and thus the payoff from R&D investment. The latter translates into an increase in R&D investment and thus an expansion in technology and TFP growth, inducing an upward shift of the trend of aggregate output. This adjustment on the long-run margin latter reduces the need for fiscal inflation. In fact, the response of growth to the shock for sufficiently small ϑ is so potent that inflation must *fall* to equate the real value of debt with the discounted present value of expected fiscal surpluses. Several features of the economy affect the inflation response to the shock, most notably ϑ and the duration of debt, ρ. The fact that said fiscal inflation can be absent in the endogenous growth economy indicates the presence of debt-stabilizing growth. That is, growth substitutes for inflation in the financing of fiscal shocks.

Figure 7: Government Spending Shock



Note: all other parameters calibrated as in Figure 1.

6. The importance of $\hat{r} - \hat{g}_y$

To the extent that growth is endogenous, the dynamics of $\hat{r} - \hat{g}_y$ are of central importance for macroeconomic stability. If the fiscal authority commits to resolving fiscal imbalances using real fiscal revenues (passive fiscal policy) then the central bank should at least ensure that $\hat{r} - \hat{g}_y$ rises with inflation. Otherwise, fiscal sustainability may require that $\hat{r} - \hat{g}_y$ falls with inflation. The *dynamics* of $\hat{r} - \hat{g}_y$ matter for stability, and the Taylor Principle is not a relevant guide to policy in the case of endogenous trend growth. Some additional discussion on these points is in order.

Blanchard (2019) among others have recently examined the costs of public debt when the rate of return on sovereign debt (r) is less than the economy's growth rate, (g_y)–a case regarded as the norm in the US by Blanchard (2019). For given r, a higher g_y improves conditions for debt stability by lowering the level of primary surpluses needed to sustain a given level of public debt. Permanent fiscal deficits can even be consistent with constant debt/GDP in cases where r is always less than g_y. This discussion clearly highlights the importance of growth for questions related to the sustainability of debt.

Our insight is fundamentally different. We show that policy should orchestrate the appropriate *changes* in $\hat{r} - \hat{g}_y$ that are needed to stabilize inflation and debt in a dynamic economy. The result does not hinge on $r - g_y < 0$ in *steady state*. In fact, our model is implicitly linearized around a steady state that features $r > g_y$ and therefore there can be no free fiscal lunch of the kind implied by: $r < g_y$. Despite the unfavorable steady state financing conditions implied by $r > g_y$, endogenous growth through R&D relaxes the conditions for fiscal sustainability under active fiscal policy relative to the analogous conditions in canonical exogenous growth model (i.e., $\partial(\hat{r} - \hat{g}_y)/\partial\hat{\pi} < 0$ versus $\partial\hat{r}/\partial\hat{\pi} < 0$). As such, a central bank that overemphasizes changes in r alone when adjusting their policy stance may inadvertently expose the economy to extraneous fluctuations. Similarly, a policymaker's preoccupation with the real interest rate might lead them to misjudge the risks of tight monetary policy for fiscal sustainability. We show it is imperative that policymakers take into account movements in r *and* short-term fluctuations in the trend growth rate, g_y, when formulating policy.

7. Conclusion

This paper studies monetary-fiscal interaction under endogenous technology growth through R&D in an otherwise standard representative agent New Keynesian model in which the real interest rate exceeds long-run growth ($r > g$). We thus depart from the previous literature which does not model long-run trend dynamics in general equilibrium.

Our key result is that endogenous trend growth alters the interaction between monetary and fiscal policy. When fiscal policy is "passive", i.e. stabilizes debt, the Taylor principle is not sufficient for determinacy. Under endogenous technology growth, an especially hawkish monetary policy may be needed to lean against an additional expectational feedback from the long-run aggregate supply side: higher expected demand raises the payoff from technological innovation and thus investment in R&D. The central bank counteracts this channel by committing to a policy that raises $r - g$ in response to a persistent increase in inflation. We show, however, that the technology growth margin relaxes conditions for debt sustainability under fiscally-unbacked ("active") fiscal policy: increased demand raises R&D and technology growth, creating additional fiscal space and alleviating inflationary pressures. We prove that active fiscal policy can be consistent with an anti-inflationary monetary policy which adheres to the Taylor principle, provided it permits a dynamic fall in $r-g$ when inflation rises persistently. In this region of the parameter space, endogenous growth dynamics back *ex-ante* unbacked fiscal deficits, *ex-post*. Taken together, these results highlight how over-emphasizing the importance of real interest rates can be misleading. Policymakers should also worry about endogenous adjustments in the long-run trend path.

These results also beg additional questions about the interaction of supply side trend developments, monetary and fiscal policy. The joint analysis of optimal monetary and fiscal policy is a particularly promising avenue in this respect which we aim to explore in future research.

References

Acharya, S. and Dogra, K. (2020). Understanding hank: Insights from a prank. *Econometrica*, 88(3):1113–1158.

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(2):323–351.

Angeletos, G.-M. and Lian, C. (2018). Forward guidance without common knowledge. *American Economic Review*, 108(9).

Angeletos, G.-M. and Lian, C. (2023). Determinacy without the taylor principle. *Journal of Political Economy*, 131(8).

Angeletos, G.-M., Lian, C., and Wolf, C. K. (2023). Can deficits finance themselves? NBER Working Paper 31185.

Antolin-Diaz, J. and Surico, P. (2022). The long-run effects of government spending. Technical report, Tech. rep., mimeographed, London Business School.

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Arrow, K. J. (1962). The Economic Implications of Learning by Doing. *Review of Economic Studies*, 29(3):155–173.

Ascari, G., Florio, A., and Gobbi, A. (2020). Controlling inflation with timid monetary–fiscal regime changes. *International Economic Review*, 61(2):1001–1024.

Beaudry, P., Hou, C., and Portier, F. (2024). Monetary policy when the phillips curve is quite flat. *American Economic Journal: Macroeconomics*, 16(1):1–28.

Benigno, G. and Fornaro, L. (2018). Stagnation traps. *The Review of Economic Studies*, 85(3):1425–1470.

Bianchi, F., Faccini, R., and Melosi, L. (2023). A fiscal theory of persistent inflation. *The Quarterly Journal of Economics*, 1438(4):2127—2179.

Bianchi, F., Kung, H., and Morales, G. (2019). Growth, slowdowns, and recoveries. *Journal of Monetary Economics*, 101:47–63.

Bilbiie, F. (2021). Monetary policy and heterogeneity: An analytical framework. Technical report.

Blanchard, O. (2019). Public debt and low interest rates. *American Economic Review*, 109(4):1197–1229.

Bonam, D. and Lukkezen, J. (2019). Fiscal and Monetary Policy Coordination, Macroeconomic Stability, and Sovereign Risk Premia. *Journal of Money, Credit and Banking*, 51(2):581–616.

Broda, C. and Weinstein, D. E. (2006). Globalization and the gains from variety. *The Quarterly Journal of Economics*, 121(2):541–585.

Brunnermeier, M., Merkel, S., and Sannikov, Y. (2022). The fiscal theory of the price level with a bubble. Technical report.

Carlstrom, C. and Fuerst, T. (2005). Investment and interest rate policy:a discrete time analysis. *Journal of Economic Theory*, 123:4–20.

Cho, S. and Moreno, A. (2021). Has fiscal policy saved the great recession? Technical report.

Cloyne, J., Martinez, J., Mumtaz, H., and Surico, P. (2022). Short-term tax cuts, long-term stimulus. NBER Working Papers 30246, National Bureau of Economic Research, Inc.

Cochrane, J. (2022). The fiscal theory of the price level. Technical report.

Comin, D. and Gertler, M. (2006). Medium-term business cycles. *American Economic Review*, 96(3):523–551.

D'Alessandro, A., Fella, G., and Melosi, L. (2019). Fiscal Stimulus With Learning-By-Doing. *International Economic Review*, 60(3):1413–1432.

Elfsbacka-Schmöller, M., Goldfayn-Frank, O., and Schmidt, T. (2023). Do recessions slow technology growth? evidence from the firm level. *Working Paper*.

Elfsbacka Schmöller, M. and Spitzer, M. (2021). Deep recessions, slowing productivity and missing (dis-) inflation in the euro area. *European Economic Review*, 134(C).

Elfsbacka Schmöller, M. (2022). Endogenous technology, scarring and fiscal policy. Bank of Finland Research Discussion Papers 13/2022, Bank of Finland.

Engler, P. and Tervala, J. (2018). Hysteresis and fiscal policy. *Journal of Economic Dynamics and Control*, 93(C):39–53.

Eusepi, S. and Preston, B. (2012). Debt, policy uncertainty and expectations stabilization. *Journal of European Economic Association*, 10(4).

Eusepi, S. and Preston, B. (2018). Fiscal foundations of inflation: Imperfect knowledge. *American Economic Review*, 108(9):2551–2589.

Fornaro, L. and Wolf, M. (2020). The scars of supply shocks. *Available at SSRN 3737556*.

Furlanetto, F., Lepetit, A., Robstad, Ø., Rubio Ramírez, J., and Ulvedal, P. (2021). Estimating hysteresis effects.

Gabaix, X. (2020). A behavioral new keynesian model. *The American Economic Review*, 110(8):2271—2327.

Galí, J. (2015). *Monetary Policy, Inflation, and the Business Cycle*. Princeton University Press.

Garga, V. and Singh, S. R. (2021). Output hysteresis and optimal monetary policy. *Journal of Monetary Economics*, 117:871–886.

Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of growth. *The Review of Economic Studies*, 58(1):43–61.

Ikeda, D. and Kurozumi, T. (2019). Slow post-financial crisis recovery and monetary policy. *American Economic Journal: Macroeconomics*, 11(4):82–112.

Ilzetzki, E. (2023). Learning by Necessity: Government Demand, Capacity Constraints, and Productivity Growth. CEPR Discussion Papers 17803, C.E.P.R. Discussion Papers.

Jordà, Ò., Singh, S. R., and Taylor, A. M. (2020). The long-run effects of monetary policy. Technical report, National Bureau of Economic Research.

Jørgensen, P. L. and Ravn, S. H. (2022). The inflation response to government spending shocks: A fiscal price puzzle? *European Economic Review*, 141(C).

Leeper, E. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.

Leeper, E. and Leith, C. (2016). Chapter 30 - understanding inflation as a joint monetary–fiscal phenomenon. volume 2 of *Handbook of Macroeconomics*, pages 2305–2415. Elsevier.

Leeper, E., Traum, N., and Walker, T. (2017). Clearing up the fiscal multiplier morass. *American Economic Review*, 107(8):2409–2454.

Ma, Y. and Zimmermann, K. (2023). Monetary Policy and Innovation. NBER Working Papers 31698, National Bureau of Economic Research, Inc.

Mehrotra, N. R. and Sergeyev, D. (2021). Debt sustainability in a low interest rate world. *Journal of Monetary Economics*, 124(S):1–18.

Micheli, M. (2018). Endogenous growth and the taylor principle. *Economics Letters*, 167:1–4.

Moran, P. and Queralto, A. (2018). Innovation, productivity, and monetary policy. *Journal of Monetary Economics*, 93:24–41.

Queraltó, A. (2022). Monetary Policy in a Model of Growth. International Finance Discussion Papers 1340, Board of Governors of the Federal Reserve System (U.S.).

Reis, R. (2022). Debt Revenue and the Sustainability of Public Debt. *Journal of Economic Perspectives*, 36(4):103–124.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102.

Sims, C. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*, 4(3):381–399.

Woodford, M. (1995). Price-level determinacy without control of a monetary aggregate. In *Carnegie-Rochester conference series on public policy*, volume 43, pages 1–46. Elsevier.

Woodford, M. (2001). Fiscal requirements for price stability. *Journal of Money, Credit, and Banking*, 33(3):669–728.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press, Princeton, NJ.

Woodford, M. and Xie, Y. (2022). Fiscal and monetary stabilization policy at the zero lower bound: consequences of limited foresight. *Journal of Monetary Economics*, 125:18–35.

A. Active Fiscal Policy Solution

This appendix provides some details about the solution of the model under active fiscal policy in the special case: $\varphi = \eta = 0$.[26] If fiscal policy is active and the GTP is violated, then there exists a unique equilibrium where inflation jumps to stabilize the real market value of detrended debt. To obtain this equilibrium, solve the government's budget constraint forward:

$$\hat{b}_{t-1} = E_t \sum_{j\geq 0} (\beta^{-1} - \gamma)^{-j-1} \left(\frac{1}{(\vartheta - 1)\beta} \hat{g}_{A,t+j-1} + \frac{1}{\beta} \hat{\pi}_{t+j} + (1 - \frac{\rho}{g_y}) \hat{P}^m_{t+j} - \frac{\bar{g}}{\bar{P}_m \bar{b}} \hat{g}_{t+j} \right). \quad (38)$$

Combining the Phillips curve (29), growth equation (26), and IS equation (28) implies a second-order difference equation in expected inflation:

$$\begin{aligned} \alpha \hat{g}_t &= E_t \hat{\pi}_{t+2} - \gamma_1 E_t \hat{\pi}_{t+1} + \gamma_0 \hat{\pi}_t \\ &= (E_t \hat{\pi}_{t+2} - \lambda_1 E_t \hat{\pi}_{t+1}) - \lambda_2 (E_t \hat{\pi}_{t+1} - \lambda_1 \pi_t), \end{aligned}$$

where $\gamma_0 := \frac{1+\tilde{c}\phi_y+\kappa\tilde{c}\phi_\pi}{\beta(1+\tilde{c}\mu)}$, $\gamma_1 := \frac{1+\beta+\beta\tilde{c}\phi_y+\kappa\tilde{c}+\tilde{c}\mu}{\beta(1+\tilde{c}\mu)}$, $\alpha := \frac{\kappa\tilde{g}\tilde{c}(\mu\rho_g-\phi_y)}{\beta(1+\tilde{c}\mu)}$, $\mu := (1-\bar{\beta})/(\vartheta-1)$, $0 < \lambda_1 := 0.5(\gamma_1 - \sqrt{\gamma_1^2 - 4\gamma_0}) < 1 < \lambda_2 := 0.5(\gamma_1 + \sqrt{\gamma_1^2 - 4\gamma_0})$. We obtain a first-order solution for expected inflation by solving the unstable root forward and stable root backward:

$$E_t \hat{\pi}_{t+j} = \lambda_1^j \hat{\pi}_t - \frac{\lambda_1^j - \rho_g^j}{(\lambda_2 - \rho_g)(\lambda_1 - \rho_g)} \alpha \hat{g}_t$$

for $j > 0$. From the last equation, $E_t \hat{\pi}_{t+1} = \lambda_1 \hat{\pi}_t - \frac{\alpha}{(\lambda_2 - \rho_g)} \hat{g}_t$. Substituting expected inflation into the Phillips curve yields the following expression for expected output:

$$E_t \hat{y}_{t+j} = \frac{1 - \beta\lambda_1}{\kappa} E_t \hat{\pi}_{t+j} + \alpha_y \rho_g^j \hat{g}_t$$

[26] The same solution approach delivers an analytical solution for $\varphi \geq 0$ and $\eta \geq 0$, but the simple case is needed for Propositions 2-3.

for $j \geq 0$, where $\alpha_y = \tilde{g} + \frac{\beta\alpha}{\kappa(\lambda_2 - \rho_g)}$. Finally, the bond price is given by:

$$\hat{P}_t^m = -E_t \sum_{s \geq 0} \left(\frac{\beta\rho}{g_y} \right)^j (\phi_\pi \pi_{t+j} + \phi_y y_{t+j}),$$

which implies that the expected bond price evolves according to

$$\hat{E}_t \hat{P}_{t+j}^m = \chi_\pi E_t \hat{\pi}_{t+j} + \chi_g \hat{g}_t.$$

Substituting the above expressions for expected inflation, output and bond price into the bond valuation equation, (38) yields closed-form solutions for endogenous variable $z \in \{\hat{\pi}, \hat{y}, \hat{i}, \hat{P}^m, \hat{b}, \hat{g}_A\}$:

$$z_t = \Omega_{z,b} \hat{b}_{t-1} + \Omega_{z,g} \hat{g}_{A,t-1} + \Gamma_z \hat{g}_t$$

In the widely studied case of a passive interest rate peg ($\phi_\pi = \phi_y = 0$) and exogenous surplus ($\gamma = 0$), the time-t inflation response to a government spending shock is given by

$$\frac{\partial \hat{\pi}_t}{\partial \hat{g}_t} = \Gamma_\pi = \xi \Omega_{\pi,b}, \tag{39}$$

$$\xi = \left(\frac{\bar{g}(g_y - \beta\rho)}{\bar{b}(1 - \beta\rho_g)} - \frac{2\beta\mu\rho_g\tilde{g}}{1 + \beta - 2\beta\rho_g + \beta(1 + \tilde{c}\mu)\sqrt{\gamma_1^2 - 4\gamma_0} + \tilde{c}(\kappa + \mu(1 - 2\beta\rho_g))} \right),$$

where

$$\Omega_{\pi,b} = \frac{\kappa(1 - \beta\lambda_1)}{\kappa + \beta\lambda_1(1 - \beta\lambda_1)\mu} \geq 0. \tag{40}$$

The expressions for $\Omega_{\pi,b}$ and Γ_π in (40) and (39) are identical to (36) and (37), respectively.

B. Proofs

B.1 Proof of Proposition 1

Case $\varphi > 0$. The model (26)-(32) can be expressed in matrix form as

$$\mathbf{Z_{t+1}} = \bar{A}\mathbf{Z_t} + \bar{C}\hat{g}_t$$

where $\mathbf{Z_{t+1}} = (E_t\hat{g}_{A,t+1}, \hat{g}_{A,t}, E_t\hat{\pi}_{t+1}, E_t\hat{y}_{t+1}, E_t\hat{P}^m_{t+1}, \hat{b}_t)'$ and

$$\bar{A} = \begin{pmatrix} \frac{\frac{\tilde{c}}{\vartheta-1}(\bar{\delta}-\eta\omega\bar{\beta})+1}{\frac{\bar{\beta}\bar{\varphi}}{1-\bar{\varphi}}} & 0 & -\frac{\tilde{c}(\bar{\delta}-\eta\omega\bar{\beta})(\beta\phi_\pi-1)}{\beta\frac{\bar{\beta}\bar{\varphi}}{1-\bar{\varphi}}} & -\frac{(\bar{\delta}-\eta\omega\bar{\beta})(\beta+\beta\tilde{c}\phi_y+\tilde{c}\kappa)+\beta\eta\omega}{\beta\frac{\bar{\beta}\bar{\varphi}}{1-\bar{\varphi}}} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 & 0 \\ -\frac{\tilde{c}}{\vartheta-1} & 0 & \tilde{c}\left(\phi_\pi - \frac{1}{\beta}\right) & \tilde{c}\left(\frac{\kappa}{\beta} + \phi_y\right) + 1 & 0 & 0 \\ 0 & 0 & \frac{g_y\phi_\pi}{\beta\rho} & \frac{g_y\phi_y}{\beta\rho} & \frac{g_y}{\beta\rho} & 0 \\ 0 & -\frac{1}{\beta(\vartheta-1)} & -\frac{1}{\beta} & 0 & \frac{\rho}{g_y} - 1 & \frac{1}{\beta} - \gamma \end{pmatrix}.$$

The model has a unique rational expectations equilibrium (REE) if 4 roots of $\bar{A}$ are outside the unit circle, a continuum of REE if fewer than 4 roots of $\bar{A}$ are outside the unit circle, and no stable solution otherwise.[27] The characteristic polynomial is:

$$\begin{aligned}
P(\lambda) &= \lambda(\lambda - (\beta^{-1} - \gamma))(\lambda - \frac{g_y}{\beta\rho})\tilde{P}(\lambda), \\
\tilde{P}(\lambda) &= \lambda^3 + p_1\lambda^2 + p_2\lambda + p_3, \\
p_1 &= -\frac{\beta(1 + C_{ga}G_{Ey}) + G_{ga}(1 + \beta + \tilde{c}(\kappa + \beta\phi_y))}{G_{ga}\beta}, \\
p_2 &= \frac{1 + C_{ga}(G_{Ey} - G_y\beta) + G_{ga} + \beta + \tilde{c}(\kappa + \beta\phi_y + G_{ga}(\phi_y + \kappa\phi_\pi))}{G_{ga}\beta}, \\
p_3 &= -\frac{1 - C_{ga}G_y + \tilde{c}(\phi_y + \kappa\phi_\pi)}{G_{ga}\beta},
\end{aligned}$$

where $G_{ga} = \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}} \in (0,1)$, $G_{Ey} = \bar{\delta} - \eta\omega\bar{\beta}$, $G_y = \omega\eta$, $C_{ga} = \frac{\tilde{c}}{\vartheta-1}$. Henceforth, we assume that $\eta < \bar{\eta}$:[28]

$$\bar{\eta} := \min\{\frac{(\vartheta-1)(1 + G_{ga} + \beta + \tilde{c}\kappa + C_{ga}\bar{\delta})}{\tilde{c}\omega(\beta + \bar{\beta})}, \frac{(\vartheta-1)(g_A - \phi + \varphi g_A(1 - \beta\bar{\beta}))}{\tilde{c}(g_A - \phi)(1 + \varphi)}\} \geq 0.$$

In turn, this implies that $p_2 > 0 > -1 > p_3$ and $p_1 < 0$, such that $\tilde{P}(\lambda) < 0$, $\forall\lambda \leq 0$, and $\lim_{\lambda\to-\infty}\tilde{P}(\lambda) = -\infty$, $\lim_{\lambda\to+\infty}\tilde{P}(\lambda) = +\infty$. Therefore, a necessary condition for all roots of $\tilde{P}(\lambda)$ to be outside the unit circle is: $\tilde{P}(1) < 0$. We have that $\tilde{P}(1) < 0$ if and only if:

$$\phi_\pi + \frac{(1-\beta)}{\kappa}\phi_y - 1 - \frac{(1-\beta)}{\kappa}\frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1 - \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)} > 0$$

From the Phillips curve, (29), $\partial\hat{y}/\partial\hat{\pi} = (1-\beta)\kappa^{-1}$, where $\partial\hat{y}/\partial\hat{\pi}$ denotes the long-run response of output to a permanent rise in inflation. Similarly, from the growth equation, (26), $\partial\hat{g}_A/\partial\hat{y} = \frac{(1-\bar{\beta})\eta\omega+\bar{\delta}}{\left(1-\frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)}$ where $\partial\hat{g}_A/\partial\hat{\pi}$ denotes the long run response of technology

[27] Following standard practices in the literature, we disregard boundary cases in which one or more roots lie on the unit circle throughout the proof of Proposition 1.

[28] To our knowledge, estimates of η are not available in the literature.

growth to a permanent rise in inflation. Therefore,

$$\begin{aligned} \phi_\pi + \frac{(1-\beta)}{\kappa}\phi_y - 1 - \frac{(1-\beta)}{\kappa}\frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1-\frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)} &= \frac{\partial(\hat{i} - \hat{\pi} - (\vartheta-1)^{-1}\hat{g}_A)}{\partial\hat{\pi}} \\ = \frac{\partial(\hat{r} - \hat{g}_y)}{\partial\hat{\pi}} &> 0 \end{aligned}$$

where $\partial\hat{g}_y/\partial\hat{\pi} = (\vartheta-1)^{-1}\partial\hat{g}_A/\partial\hat{\pi}$ is the response of trend output growth to a permanent rise in inflation. Hence, if fiscal policy is passive ($|\beta^{-1} - \gamma| < 1$) then a necessary condition for determinacy is $\frac{\partial(\hat{r}-\hat{g}_y)}{\partial\hat{\pi}} > 0$.

Now suppose that $\tilde{P}(1) > 0$, such that $\frac{\partial(\hat{r}-\hat{g}_y)}{\partial\hat{\pi}} < 0$. Then one real root, λ_1, of $\tilde{P}(\lambda)$ is inside the unit circle and strictly positive. Let λ_2, λ_3 denote the remaining roots, and recall that any real roots cannot be negative. Then: $-\lambda_1\lambda_2\lambda_3 = p_3 < -1$. Therefore: $\lambda_2\lambda_3 = |p_3|/\lambda_1 > 1$. If the remaining roots are complex, then $\lambda_2\lambda_3 = |\lambda_2| = |\lambda_3| > 1$. If the remaining roots are real, then without loss of generality, $\lambda_2 > 1$, which implies $\lambda_3 > 1$ because $\tilde{P}(0) < 0 < \tilde{P}(1)$ implies a maximum of one real root inside the unit circle if $\lambda_2 > 1 > \lambda_1 > 0$ and all roots are real and non-negative. We conclude that $\tilde{P}(1) > 0$ is sufficient for determinacy under active fiscal policy.

Case $\varphi = 0$. If $\varphi = 0$ then the relevant characteristic polynomial becomes:

$$\begin{aligned} Q(\lambda) &= \lambda(\lambda - (\beta^{-1} - \gamma))(\lambda - \frac{g_y}{\beta\rho})\tilde{P}(\lambda), \\ \tilde{Q}(\lambda) &= \lambda^2 + q_1\lambda + q_2, \\ q_1 &= -\frac{(1 + C_{ga}(G_{Ey} - \beta G_y) + \beta + \tilde{c}(\kappa + \beta\phi_y))}{\beta(1 + C_{ga}G_{Ey})}, \\ q_2 &= \frac{1 - C_{ga}G_y + \tilde{c}(\phi_y + \kappa\phi_\pi)}{\beta(1 + C_{ga}G_{Ey})}. \end{aligned}$$

The model with $\varphi = 0$ has a unique rational expectations equilibrium (REE) if 3 roots of $Q(\lambda)$ are outside the unit circle, a continuum of REE if fewer than 3 roots of $Q(\lambda)$ are outside the unit circle, and no stable solution otherwise. Under passive (active) fiscal policy, determinacy therefore requires that two (one) root(s) of $\tilde{Q}(\lambda)$ are outside the unit circle. If $\eta < \bar{\eta}$ then $\tilde{Q}(0) = q_2 > 0 > q_1$ and $Q(\lambda) > 0$ for $\lambda \leq 0$. It follows that $\tilde{Q}(1) > 0$

is a necessary condition for determinacy under passive fiscal policy which is satisfied if and only if:

$$\phi_\pi + \frac{(1-\beta)}{\kappa}\phi_y - 1 - \frac{(1-\beta)}{\kappa}\frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{\vartheta - 1} = \frac{\partial(\hat{r} - \hat{g}_y)}{\partial \hat{\pi}} > 0$$

If $\frac{\partial(r-g_y)}{\partial\hat{\pi}} < 0$ then a real root, λ_1, is in $(0,1)$. Because $\tilde{Q}(1) < 0$ and $\lim_{\lambda\to+\infty}\tilde{Q}(\lambda) = +\infty$ the other root, λ_2, is strictly greater than one. Therefore, $\frac{\partial(\hat{r}-\hat{g}_y)}{\partial\hat{\pi}} < 0$ is sufficient for determinacy under active fiscal policy.

B.2 Proof of Proposition 2

Consider (37) and (36). If $\vartheta = \infty$ (equivalently, $\mu = (1-\bar{\beta})/(\vartheta - 1) = 0$) then:

$$\begin{aligned} \Gamma_\pi &= \Omega_{\pi,b}\left(\frac{\bar{g}(g_y - \beta\rho)}{\bar{b}(1-\beta\rho_g)}\right), \\ \Omega_{\pi,b} &= 1 - \beta\lambda_1 > 0, \end{aligned}$$

which proves the assertions.

B.3 Proof of Proposition 3

Consider (37) and (36). From (36), $\Omega_{\pi,b} > 0$ (with strict inequality) if $\vartheta > 1$ ($\mu < \infty$). Therefore, $\Gamma_\pi \leq 0$ if and only if $\xi \leq 0$. Clearly, ξ is strictly decreasing in ρ, and there exists a unique ρ^* such that $\xi = 0$ if and only if $\rho = \rho^*$.[29]

C. Model Derivation Details

C.1 Stationarized equilibrium conditions

Given the presence of positive trend growth in the economy, we express the equilibrium conditions in terms of the following stationary variables: $\{\mathcal{J}_t, \mathbf{\Pi}_t, y_t, c_t, g_t, w_t, w_t^s, b_t, \tilde{T}_t\}$

[29] It is assumed that $\bar{g} \neq 0$.

$:= \{ \frac{J_t}{A_t^{\frac{2-\vartheta}{\vartheta-1}}}, \frac{\Pi_t}{A_t^{\frac{2-\vartheta}{\vartheta-1}}}, \frac{Y_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{C_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{G_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{W_t/P_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{W_t^s/P_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{B_t/P_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{T_t}{A_t^{\frac{1}{\vartheta-1}}} \}$. The model equilibrium conditions expressed in terms of stationary variables are given by:

$$g_{A,t} = \phi + \varsigma \left(L_t\right)^{\eta} L_t^s \tag{C.1}$$

$$\tilde{J}_t = E_t \left\{ \frac{\beta}{g_{A,t}} \frac{c_t}{c_{t+1}} \left(\tilde{\Pi}_{t+1} + \phi \tilde{J}_{t+1} \right) \right\} \tag{C.2}$$

$$\varsigma \left(L_t\right)^{\eta} \tilde{J}_t = w_t^s \tag{C.3}$$

$$g_{y,t} = \left(g_{A,t}\right)^{\frac{1}{\vartheta-1}} \tag{C.4}$$

$$\tilde{\Pi}_t = \vartheta^{-1} MC_t y_t \nu_t \tag{C.5}$$

$$MC_t = \frac{\vartheta}{\vartheta-1} w_t \tag{C.6}$$

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} c_{t+j}^{-1} \beta^j \theta^j \left(\frac{P_t^*}{P_{t+j}} - \frac{\epsilon}{\epsilon-1} MC_{t+j} \right) \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} y_{t+j} \right\} = 0 \tag{C.7}$$

$$R_t = r\pi^* \left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}} \right)^{\phi_y} \tag{C.8}$$

$$b_t P_t^m + \tilde{T}_t = b_{t-1} \frac{(1+\rho P_t^m)}{\pi_t g_{y,t-1}} + g_t \tag{C.9}$$

$$\frac{\tilde{T}_t}{\bar{\tilde{T}}} = \left(\frac{b_{t-1}}{\bar{b}} \right)^{\frac{\gamma \bar{P}^m \bar{b}}{\bar{\tilde{T}}}} \tag{C.10}$$

$$1 = \beta \mathbb{E}_t \left\{ \frac{c_t}{c_{t+1}} \frac{R_t}{g_{y,t} \pi_{t+1}} \right\} \tag{C.11}$$

$$P_t^m = E_t \left\{ R_t^{-1} \left(1 + \rho P_{t+1}^m \right) \right\} \tag{C.12}$$

$$(L_t)^{\varphi} c_t = w_t \tag{C.13}$$

$$\chi (L_t^s)^{\varphi} c_t = w_t^s \tag{C.14}$$

$$\frac{g_t}{\bar{g}} = \left(\frac{g_{t-1}}{\bar{g}} \right)^{\rho_g} \epsilon_t^G \tag{C.15}$$

$$y_t = \nu_t^{-1} L_t \tag{C.16}$$

$$y_t = c_t + g_t \tag{C.17}$$

$$P_t = \left[(1-\theta) \left(P_t^*\right)^{1-\epsilon} + \theta \left(P_{t-1}\right)^{1-\epsilon)} \right]^{\frac{1}{1-\epsilon}} \tag{C.18}$$

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{C.19}$$

where $\nu_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di$. In an equilibrium, the endogenous variable vector:

$$\{c_t, y_t, g_t, L_t, L_t^s, g_{A,t}, g_{y,t}, \mathbf{\Pi}_t, MC_t, \mathcal{J}_t, b_t, \tilde{T}_t, P_t^m, R_t, P_t, P_t^*, \pi_t, w_t, w_t^s\}$$

must satisfy the equilibrium conditions (C.1)-(C.19) given the government spending shocks $\{\epsilon_t^G\}$ and initial conditions, b_{-1}, P_{-1}.

C.2 Steady state

Let $\bar{Z}$ denote the steady state value of variable Z. Note that we define government spending policy such that $\bar{G}/\bar{Y} = \tilde{g} > 0$ and $\bar{C}/\bar{Y} = 1 - \tilde{g} = \tilde{c}$. From (C.7), the steady state marginal cost is given by

$$\bar{MC} = \frac{\epsilon - 1}{\epsilon}$$

Combining this expression for steady state marginal cost, $\bar{MC}$, with (C.6), (C.13) and (C.16) yields:

$$\bar{L} = \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi+1}}$$

and therefore, from (C.5), (C.13), and (C.16):

$$\begin{aligned}
\bar{y} &= \bar{L}\nu = \nu\left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi+1}} \\
\bar{c} &= \tilde{c}\bar{y} = \tilde{c}\nu\left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi+1}} \\
\bar{g} &= \tilde{g}\bar{y} = \tilde{g}\nu\left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi+1}} \\
\bar{\mathbf{\Pi}} &= \frac{\bar{MC}}{\vartheta}\bar{L} = \frac{\epsilon - 1}{\vartheta\epsilon}\left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi+1}}
\end{aligned}$$

Given, $\bar{L}$, $\bar{\mathbf{\Pi}}$, and $\bar{c}$ from above, $\bar{\mathcal{J}}$, $g_a := \frac{\bar{A_{t+1}}}{A_t}$, $\bar{L}^s$ jointly satisfy (C.1)-(C.3) after substituting in (C.14):

$$\begin{aligned}
\bar{\mathcal{J}} &= \frac{\beta}{g_A - \beta\phi}\bar{\mathbf{\Pi}} \\
g_A &= \phi + \varsigma(\bar{L})^{\eta}\bar{L}^s \\
\chi\bar{c}(\bar{L}^s)^{\varphi} &= \varsigma\bar{\mathcal{J}}(\bar{L})^{\eta}
\end{aligned}$$

We proceed by calibrating $g_A \geq 1$ (e.g. to match the trend output growth rate, $g_y = g_A^{\frac{1}{\vartheta-1}}$, in the economy) by solving the last three equations for $\bar{\mathcal{J}}, \bar{L}^s, \varsigma$, where ς scales the marginal product of skilled labor in R&D production.[30] If $\varphi > 0$:

$$\begin{aligned}
\varsigma &= \left((g_A - \phi)\left(\frac{\nu(\epsilon-1)(\vartheta-1)}{\tilde{c}\epsilon\vartheta}\right)^{\frac{-\eta}{\varphi+1}} \left(\frac{\beta\left(\frac{\nu(\epsilon-1)(\vartheta-1)}{\tilde{c}\epsilon\vartheta}\right)^{\frac{1+\eta+\varphi}{\varphi+1}}}{\chi(\vartheta-1)(g_A-\beta\phi)} \right)^{-1/\varphi} \right)^{\frac{\varphi}{\varphi+1}} > 0 \\
\bar{L}^s &= \varsigma^{\frac{1}{\varphi}} \left(\frac{\beta\left(\frac{\nu(\epsilon-1)(\vartheta-1)}{\tilde{c}\epsilon\vartheta}\right)^{\frac{1+\eta+\varphi}{\varphi+1}}}{\chi(\vartheta-1)(g_A-\beta\phi)} \right)^{\frac{1}{\varphi}} \\
\bar{\mathcal{J}} &= \frac{\beta(\epsilon-1)}{(g_A-\beta\phi)\vartheta\epsilon}\left(\frac{\nu(\vartheta-1)(\epsilon-1)}{\tilde{c}\vartheta}\right)^{\frac{1}{\varphi+1}}
\end{aligned}$$

Otherwise, if $\varphi = 0$:

$$\begin{aligned}
\varsigma &= \frac{\tilde{c}\chi\epsilon\vartheta(g_A-\beta\phi)\left(\frac{\nu(\epsilon-1)}{\tilde{c}\epsilon\vartheta}\right)^{-\eta}}{\beta\nu(\epsilon-1)(\vartheta-1)} > 0 \\
\bar{L}^s &= \frac{\beta\nu(\epsilon-1)(g_A-\phi)}{\tilde{c}\chi\epsilon\vartheta(g_A-\beta\phi)} \\
\bar{\mathcal{J}} &= \frac{\beta(\epsilon-1)}{(g_A-\beta\phi)\vartheta\epsilon}\left(\frac{\nu(\vartheta-1)(\epsilon-1)}{\tilde{c}\vartheta}\right)
\end{aligned}$$

[30] One alternative is that we solve these equations for $\bar{\mathcal{J}}, \bar{L}^s, \chi$ given ς. That is, we may back out a chosen g_A by adjusting the parameter χ (which affects marginal disutility of skilled labor). Another option is that we calibrate both χ and ς and solve these equations for $\bar{\mathcal{J}}, \bar{L}^s, g_A$.

From (C.11) and (C.12):

$$\begin{aligned}\bar{R} &= r\pi^* = \beta^{-1} g_y \\ \bar{P}^m &= \frac{\beta}{g_y - \beta\rho}\end{aligned}$$

where $g_y = (g_A)^{\frac{1}{\vartheta - 1}}$. We calibrate $\bar{b} = dy(\bar{y})$ where dy is the debt-to-GDP ratio and solve for $\bar{\tilde{T}}$ that satisfies the government's intertemporal budget constraint (C.9):

$$\bar{\tilde{T}} = \bar{b}\left(\frac{1 + \rho\bar{P}^m}{g_y} - \bar{P}^m\right) + \bar{g} = \bar{b}\left(\frac{1-\beta}{g_y - \beta\rho}\right) + \bar{g} > 0.$$

where $\bar{g}$, $\bar{P}^m$, g_y are defined in terms of deep structural parameters above. At this stage, it is possible to note that the coefficient multiplying $\hat{g}_t$ in (31) can be expressed as $\bar{g}/(\bar{P}^m\bar{b}) = \tilde{g}(g_y - \beta\rho)/(\beta dy)$. Finally, $\bar{\nu} = \pi^* = 1$ in steady state, and $\bar{w}$, $\bar{w}^s$ are given by (C.13)-(C.14) after substituting for $\bar{L}, \bar{L}^s, \bar{c}$. Therefore, we can derive the steady state for the stationarized variables: $(\bar{c}, \bar{y}, \bar{g}, \bar{L}, \bar{L}^s, g_A, g_y, \bar{\mathbf{\Pi}}, \bar{MC}, \bar{\mathcal{J}}, \bar{b}, \bar{\tilde{T}}, \bar{P^m}, \bar{R}, \pi^*, \bar{w}, \bar{w}^s)$.

C.3 Linearization details

The model is log-linearized at the steady state described above.[31] Equation (26) is obtained by log-linearizing (C.1)-(C.3), (C.5)-(C.6), (C.13)-(C.14) and (C.16) and combining the resulting expressions. Equation (27) is obtained from (C.4). Equation (28) is obtained from (C.11), (C.15), and (C.17). Equation (29) is obtained from (C.6)-(C.7), (C.13), (C.15)-(C.19). Equation (30) is obtained from (C.8). Equation (31) is obtained from (C.9)-(C.10) and (C.15). Equation (32) is obtained from (C.12).

[31] Recall that we assume $\tilde{g} > 0$ and $\bar{b} > 0$.