Does my model predict a forward guidance puzzle?

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Abstract

We provide sufficient conditions for when a rational expectations structural model predicts bounded responses of endogenous variables to forward guidance announcements. The conditions coincide with a special case of the well-known (E)xpectation-stability conditions that govern when agents can learn a Rational Expectations Equilibrium. Importantly, we show that the conditions are distinct from the determinacy conditions. We show how the conditions are useful for diagnosing the features of a model that contribute to the Forward Guidance Puzzle and reveal how to construct well-behaved forward guidance predictions in standard medium-scale DSGE models.

JEL Classifications: E31; E32; E52; D84; D83.

Key Words: Forward Guidance Puzzle; Expectations; Learning; Stability.

1 Introduction

A near ubiquitous feature of standard rational expectations (RE) structural monetary policy models is that credible promises to hold interest rates at zero for extended periods of time can generate significant jumps in output and inflation in the period the policy is announced. Moreover, the contemporaneous impact of such policies can be made arbitrarily large today simply by pushing the actual implementation of the policy farther into the future, a phenomenon known as the *Forward Guidance Puzzle*. Since this feature

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of structural monetary models was first pointed out by papers such as Del Negro et al. (2012) and Carlstrom et al. (2015), a number of authors have sought to ameliorate and explain away this puzzle using, for example, credibility (Haberis et al., 2019), imperfect information (Carlstrom et al., 2015; Kiley, 2016), bounded rationality (Gabaix, 2016; Angeletos and Lian, 2018), life-cycle considerations (Del Negro et al., 2012; Eggertsson and Mehrotra, 2014; Eggertsson et al., 2019), heterogeneous agents with incomplete markets (McKay et al., 2016), or the fiscal theory of the price level (Cochrane, 2017; McClung, 2019) to name just a few.

In this paper, we provide sufficient conditions for when forward guidance announcements will have bounded contemporaneous impacts in standard structural models, including in models of Markov-switching, and hence do not display the Forward Guidance Puzzle. The conditions turn out to be a special case of the E-stability conditions popularized by Evans and Honkapohja (2001) known as (I)terative E-stability. E-stability has a long history in macroeconomics as an equilibrium selection device. Traditionally, E-stability is used to select Rational Expectations Equilibrium (REE) that are 'learnable' by agents who may use a variety of econometric techniques to infer the solution from past data. REEs that are E-stable are argued to be more plausible than those that are not because coordination on rational beliefs can be cast as an endogenous outcome arising from less sophisticated behavior.[1]

IE-stability is a stricter version of E-stability that is often associated with the concept of rationalizability in games.[2] Evans and Guesnerie (1993) and Guesnerie (2002), for example, shows that IE-stability is closely related to the notion of rationalizability when considering coordination on an REE in dynamic macro models among rational agents

[1]E-stability is widely used in monetary economics as an equilibrium selection such as in Bullard and Mitra (2002) and Christiano et al. (2018).

[2]IE-stability actually predates the E-stability concept. Evans (1983), Evans (1985), and Evans (1986) all propose iterative procedures as an equilibrium selection criterion, which Evans (1985) defined as "Expectational Stability." Later, Evans (1989) showed that this criterion shared features with the convergence conditions for agents who engage in least squares learning studied in Marcet and Sargent (1989). Further contrasts between the two approaches are discussed in Evans and Honkapohja (1992) and the current terminology of E-stability and IE-stability dates back to Evans and Honkapohja (1994).

who possess the common knowledge of rationality. The connection to rationalizability means that one way to view IE-stability in relation to forward guidance announcements is as selecting the "plausible" equilibrium predictions. This interpretation aligns well with the analysis of Cochrane (2017) for generating well-behaved zero interest rate forward guidance policies since IE-stability can select Cochrane's preferred equilibrium in some instances. But this interpretation conflicts with García-Schmidt and Woodford (2019) conclusions that the assumptions underpinning IE-stable solutions are too strong.

For the purposes of this paper, we do not take a position on which equilibria are more appealing on theoretical grounds. Instead, we focus on the fact that IE-stability reduces the assessment of forward guidance to studying the properties of eigenvalues recovered from key structural matrices of the economy implied by a given forward guidance announcement. IE-stability is a useful tool to diagnose features of a model that contribute to the puzzle. Our analysis, therefore, is an extension and formalization of the eigenvalue analysis done by Carlstrom et al. (2015) in their study of forward guidance. We illustrate how IE-stability analysis is useful in some of the aforementioned models listed above.

Four additional aspects of the Forward Guidance Puzzle are illuminated by this analysis. First, there are actually many Forward Guidance Puzzles in that any anticipated policy change may result in similar predictions to interest rate forward guidance when IE-stability of the announcement is not satisfied. This means, for example, that IE-stability conditions are predictive for the forward fiscal guidance puzzle explored by Canzoneri et al. (2018), or for when large positive/negative impacts of announced disinflation arise such as in Ball (1994) and Gibbs and Kulish (2017). Second, IE-stability also diagnoses other so-called policy paradoxes at the zero lower bound (ZLB), such as the paradox of toil and paradox of volatility (Eggertsson, 2010 and Kiley, 2016). Third, the Forward Guidance Puzzle is not solely a product of indeterminacy as suggested by Carlstrom et al. (2015). It just happens to be the case that determinacy often implies IE-stability for a wide class of models.[3] We provide a specific example of this in a regime-switching DSGE model with fiscal and monetary policy that is indeterminate under an interest rate peg but satisfies our IE-stability conditions. Here, despite indeterminacy, we find well-behaved contemporaneous responses to forward guidance announcements.

We also show that the Forward Guidance Puzzle is not just model dependent but also policy dependent. The dependence follows immediately from the fact that IE-stability is only a locally sufficient condition in most models. Therefore, depending on the nature of the announcement both bounded and unbounded policy responses for fixed interest rate announcements are possible. We illustrate this by studying forward guidance in the model of Smets and Wouters (2007). We show that the Forward Guidance Puzzle in this case depends on whether agents believe that monetary policy returns to an active or a passive regime and on the existence of sunspots. If policy is active upon the lift off of interest rates, as is usually assumed, then the standard forward guidance predictions are found. If, however, the policy is passive upon lift off and agents coordinate on a sunspot, then the amount of stimulus provided by any duration of the announcement is bounded.

Finally, we show how the intuition of the IE-stability conditions can be applied to models of sticky information.

2 Forward guidance in structural models

To fix ideas, let us consider linear structural models that take the following form

$$y_t = \Gamma(\theta) + A(\theta)y_{t-1} + B(\theta)\mathbb{E}_t y_{t+1} + D(\theta)\omega_t \quad (1)$$

$$\omega_t = \rho(\theta)\omega_{t-1} + \varepsilon_t \quad (2)$$

[3]The connection between determinacy and E-stability has been widely studied, for example, in McCallum (2007) and Bullard and Eusepi (2014). Ellison and Pearlman (2011) also extends this analysis to IE-stability.

where y_t is a $n \times 1$ vector of endogenous variables, ω_t is $k \times 1$ vector of vector of exogenous variables, ε_t is a vector of exogenous white noise innovations, and θ is a vector of deep parameters that define agent and policymaker's behavior. We restrict our attention to stationary RE solutions, but we do not impose that they are unique.

Formally, we define a forward guidance announcement, its impact, and the Forward Guidance Puzzle as follows:

Definition 1: *A Forward Guidance Announcement (FGA) is a set* $\{\theta_i\}_{i=T^a,T^*}$ *such that* $T^* - T^a = \Delta_p > 0$*, where* θ_{T^a} *is the vector of structural parameters that governs the economy from time of the announcement,* T^a*, until time* $T^* - 1$*.* θ_{T^*} *is the vector of structural parameters that governs the economy at time* $t = T^*$*, when the policy ends.*

Definition 2: *The contemporaneous impact of an FGA is defined as* $|y_{ss} - \mathbb{E}[y_{T^a}]|$*, where the* $\mathbb{E}[y_{T^a}]$ *is the unconditional expectation of the vector of endogenous variables at time* $t = T^a$ *and* y_{ss} *is the steady state of the model when* $t < T^a$*.*

Definition 3: *An FGA* $\{\theta_i\}_{i=T^a,T^*}$ *is said to exhibit The Forward Guidance Puzzle (FGP) if its impact is unbounded as* $\Delta_p \to \infty$*.*

Our FGA definition makes two important simplifications to make the analysis clearer and which are easily generalized. The first simplification is that elements ε_t are not included in the θ_i's. Therefore, for example, anticipated monetary policy shocks are modeled as temporary changes in the intercept of the policy rule rather than known realizations of ε in the future.[4]

[4] Arguably, temporary changes to an intercept is the most logically consistent way to model such an FGA since the policymakers in most RE models share the private sector agents' information set, which takes the policy shocks as exogenous. From our analysis it will be clear how the same conditions apply to anticipated shocks.

The second simplification is that all FGA's are reduced to two regimes. It is straightforward to reduce more complicated FGA's though to fit into our framework. For example, the standard forward guidance thought experiment typically has three parts: 1) a suspension of active policy for some duration Δ_p, 2) a one-time anticipated monetary policy shock occurring in period T^*, 3) and active policy resuming thereafter. The suspension of active policy, the policy shock, and the resumption of policy are three separate regimes that can be represented by three different θ_i's. However, what matters for assessing the impact of an FGA is the RE solution in period $t = T^*$. The RE solution at $t = T^*$ will be different depending on how active policy is implemented in $T^* + 1$, but that difference is completely encoded in the T^* solution. So, once the solution at time T^* is known, all that matter is its relationship to the T^a regime. In this way, it always possible to collapse any FGA into a two-regime problem for the purposes of assessing its impact.

2.1 RE solutions to forward guidance announcements

The most common strategy for obtaining the RE solution to an FGA is to use the method of undetermined coefficients combined with backward induction. For example, this is the approach taken by Eggertsson and Woodford (2003) to study optimal policy at the ZLB. It is also the approach that underpins the solution method for anticipated structural changes in Cagliarini and Kulish (2013) and Kulish and Pagan (2017). Similar approaches are also employed in Cho and Moreno (2011) and to solve for RE solutions of Markov-switching DSGE models as in Baele et al. (2015) and Cho (2016).

The method typically proceeds as follows. For convenience, we assume that θ_{T^*} describes the economy for all $t \geq T^*$. In period $t = T^*$, the stationary minimum state variable (MSV) RE solution takes the form of

$$y_t = a + by_{t-1} + c\omega_t \tag{3}$$

This implies that the expectation of y_{t+1} in time t is given by

$$\mathbb{E}_t y_{t+1} = a + by_t + c\rho(\theta_{T^*})\omega_t \tag{4}$$

Substituting equation (4) into equation (1), we have

$$y_t = (I - B(\theta_{T^*})b)^{-1} \left(\Gamma(\theta_{T^*}) + B(\theta_{T^*})a + (B(\theta)c\rho(\theta_{T^*}) + D(\theta_{T^*}))\,\omega_t + A(\theta_{T^*})y_{t-1}\right). \tag{5}$$

Equating equation (5) with (3) we arrive at the following equivalences

$$\begin{aligned} a &= (I - B_* b)^{-1} (\Gamma_* + B_* a) && (6)\\ b &= (I - B_* b)^{-1} A_* && (7)\\ c &= (I - B_* b)^{-1} (B_* c\rho_* + D_*)) && (8) \end{aligned}$$

where to simplify notation we write $B_* = B(\theta_{T^*})$ and $B_a = B(\theta_{T^a})$, etc. The MSV RE solution at time $t = T^*$ is given by $\bar{a}(\theta_{T^*})$, $\bar{b}(\theta_{T^*})$, and $\bar{c}(\theta_{T^*})$, which satisfy equations (6), (7), and (8).

In period $t = T^* - 1$, the MSV solution again takes the same form as equation (3). Expectations of y_{t+1} at time t, however, are no longer unknown. They are given by

$$\mathbb{E}_t y_{t+1} = \bar{a}(\theta_{T^*}) + \bar{b}(\theta_{T^*})y_t + \bar{c}(\theta_{T^*})\rho_a\omega_t$$

Substituting expectations into equation (1) and equating with equation (3) , we now have the following equivalences

$$\begin{aligned} a &= \left(I - B_a\bar{b}(\theta_{T^*})\right)^{-1} (\Gamma_a + B_a\bar{a}(\theta_{T^*}))\\ b &= \left(I - B_a\bar{b}(\theta_{T^*})\right)^{-1} A_a\\ c &= \left(I - B_a\bar{b}(\theta_{T^*})\right)^{-1} (B_a\bar{c}(\theta_{T^*})\rho_a + D_a)\,, \end{aligned}$$

which defines the RE solution for $t = T^* - 1$. Continuing to work backwards in time, the entire RE solution for the FGA can be written recursively. To illustrate this, define j as the number of periods remaining until T^* (i.e. $j = T^* - t$), which allows us to write the RE solution as

$$\bar{a}_j = (I - B_a \bar{b}_{j-1})^{-1} (\Gamma_a + B_a \bar{a}_{j-1}) \tag{9}$$

$$\bar{b}_j = (I - B_a \bar{b}_{j-1})^{-1} A_a \tag{10}$$

$$\bar{c}_j = (I - B_a \bar{b}_{j-1})^{-1} (B_a \bar{c}_{j-1} \rho_a + D_a)) \tag{11}$$

where $\bar{a}_0 = \bar{a}(\theta_{T^*})$, $\bar{b}_0 = \bar{b}(\theta_{T^*})$, and $\bar{c}_0 = \bar{a}(\theta_{T^*})$.

2.2 Connection to IE-stability

Equations (9), (10), and (11) should look familiar to anyone who has studied a model under adaptive learning. This is because under adaptive learning we typically start with the assumption that agents form time t expectations using estimates of the MSV RE solution based on all data up to time $t - 1$ such that

$$\mathbb{E}_t y_{t+1} = a_{t-1} + b_{t-1} y_t + c_{t-1} \rho \omega_t.$$

As before, beliefs are substituted into equation (1) to find the actual law of motion for the economy

$$y_t = (I - B b_{t-1})^{-1} (\Gamma + B a_{t-1} + (B c_{t-1} \rho + D) \omega_t + A y_{t-1}).$$

Notice that the actual law of motion reveals the same mapping from agents' beliefs about the MSV RE solution coefficients, $\phi_{t-1} = (a_{t-1}, b_{t-1}, c_{t-1})$, to the actual equilibrium coefficients as derived previously. This mapping is known as the T-map, where

$$T(\phi_{t-1}) = \left((I - Bb_{t-1})^{-1}(\Gamma + Ba_{t-1}), (I - Bb_{t-1})^{-1}A, (I - Bb_{t-1})^{-1}(Bc_{t-1}\rho + D)\right).$$

The T-map plays a crucial rule in the adaptive learning literature, and we can express (9), (10), and (11) equivalently as $\phi_t = (a_t, b_t, c_t) = T(\phi_{t-1})$, or:

$$\begin{aligned} a_t &= (I - Bb_{t-1})^{-1}(\Gamma + Ba_{t-1}) \\ b_t &= (I - Bb_{t-1})^{-1}A \\ c_t &= (I - Bb_{t-1})^{-1}(Bc_{t-1}\rho + D). \end{aligned}$$

Hence, we obtain RE solutions for the FGA by iterating on the T-map.

Definition 4: *A fixed point of the T-map, $\bar{\phi}$, is said to be Iteratively E-stable if for all ϕ_0 in a neighborhood of $\bar{\phi}$,*

$$\phi_N \to \bar{\phi}$$

as $N \to \infty$.

Theorem 1 (Restatement of 10.3 Evans and Honkapohja, 2001) *An MSV solution $\bar{a}$, $\bar{b}$, and $\bar{c}$ is IE-stable if all eigenvalues of*

$$\begin{aligned} DT_a(\bar{a}, \bar{b}) &= (I - B\bar{b})^{-1}B \\ DT_b(\bar{b}) &= \left[(I - B\bar{b})^{-1}A\right]' \otimes \left[(I - B\bar{b})^{-1}B\right] \\ DT_c(\bar{b}, \bar{c}) &= \rho' \otimes \left[(I - B\bar{b})^{-1}B\right] \end{aligned}$$

have modulus less than 1. The solution is not IE-stable if any of the eigenvalues have modulus larger than 1.

The IE-stability condition is the local stability condition for a fixed point of a system of nonlinear difference equations. The Forward Guidance Puzzle, therefore, can be viewed as resulting from non-convergence of the system of equations given by (9), (10), and (11) from the terminal solution defined by θ_{T^*}. We summarize this connection as a proposition.

Proposition 1 *The impact of a FGA* $\{\theta_i\}_{i=T^a,T^*}$ *is bounded as* $\Delta_p \to \infty$ *if*

1. $\bar{\phi}(\theta_{T^a})$ *exists*

2. $\bar{\phi}(\theta_{T^a})$ *is IE-stable*

3. *and* $\phi_0(\theta_{T^*})$ *is in the appropriate neighborhood of* $\bar{\phi}(\theta_{T^a})$

The proof of Proposition 1 is in the appendix. Proposition 1 is only a sufficient condition because it does not rule out cases where the recursion results in bounded non-converging cycles. These cycles would also generate bounded impacts.[5] The economic relevance of such cases is debatable and the range of the parameter space over which they could occur in most models is small so Proposition 1 functions as a necessary condition for the majority of economically relevant scenarios.

There are three important additional remarks. First, the proposition requires that $\bar{\phi}(\theta_{T^a})$ exists but does not require that it is unique. In fact, the canonical forward guidance experiment of an interest rate peg implies indeterminacy in most monetary policy models. When this occurs, the precise $\bar{\phi}(\theta_{T^a})$ that the recursion settles on, if it converges, may depend on $\phi_0(\theta_{T^*})$. Second, the proposition depends explicitly on $\phi_0(\theta_{T^*})$. Even if $\bar{\phi}(\theta_{T^a})$ exists and is IE-stable, the FGA may not have a bounded impact if $\phi_0(\theta_{T^*})$ is not in the appropriate basin of attraction. Therefore, the same θ_{T^a} regime can generate bounded and unbounded impacts for different $\phi_0(\theta_{T^*})$. The only time this dependence on the terminal regime is broken is in the case when the recursions are linear.

[5]The cycles we refer to here are distinct from those discussed in Carlstrom et al. (2015). Those cycles are not bounded and are ruled out by the proposition.

Corollary *When there are no lagged endogenous variables, $A = 0$, the impact of an FGA $\{\theta_i\}_{i=T^A,T^*}$ is bounded as $\Delta_p \to \infty$ if $\bar{\phi}(\theta_{T^a})$ is IE-stable.*

The third remark is that McCallum (2007) shows that determinacy implies E-stability in a wide class of linear RE models under the same assumptions we use to derive E-stability conditions in this paper. However, E-stability does not imply determinacy. The two are distinct concepts and there exist indeterminate RE solutions, which are E-stable. Ellison and Pearlman (2011) extends McCallum's analysis to IE-stability and shows that there are IE-stable indeterminate equilibria within a certain class of linear RE models. Therefore, indeterminacy is not a sufficient condition for Forward Guidance Puzzle like behavior to be observed. We provide a specific and economically relevant example of this distinction in a Markov-switching model in Section 4.

2.3 An aside on "backward-stability"

We note that our IE-stability criterion selects a "backward-stable" equilibrium in the sense of Cochrane (2017) whenever the conditions in Proposition 1 are satisfied. In his paper, Cochrane calls the equilibria of an indeterminate New Keynesian model that features well-behaved impacts of forward guidance "backward-stable". This is in contrast to the usual forward stable determinate solutions.

Our approach differs from Cochrane (2017) in an important way: Cochrane imposes explicit initial conditions (e.g. no response of inflation to forward guidance on impact) to select a backward-stable equilibrium, whereas our method always starts from a terminal condition, $\phi_0(\theta_{T^*})$. This means that the solutions that IE-stability selects are pinned down in part by the characteristics of the policy itself and not by an imposed restriction by the modeler on market behavior. Furthermore, one can appeal to IE-stability's connection to rationalizability as a plausible reason to rule out or select one predicted outcome over another.

A particularly salient example is the simple New Keynesian model with an interest rate peg. Here, there exists no IE-stable MSV solutions for any $\phi_0(\theta_{T^*})$ because it lacks any lagged endogenous variable as in our Corollary. However, as Cochrane points out, we can find backwardly stable equilibria in this environment if we are willing to augment the model's system of equations to explicitly include other equilibrium conditions such as the government debt valuation in Cochrane (2017). Similarly, the same result may be achieved if we allow agents to coordinate on explicit sunspot solutions. Implicitly, government debt and sunspots are always part of the simple New Keynesian environment but are rendered irrelevant when monetary policy takes an active stabilization role. The consideration of these features reintroduces dependence on $\phi_0(\theta_{T^*})$ for forward guidance announcements. In this environment, IE-stability allows us to select backward-stable solutions as part of the specification of policy by encoding these considerations in the terminal belief, $\phi_0(\theta_{T^*})$. Therefore, whether a forward guidance announcement predicts a puzzling result depends explicitly on the specification of policy in our framework.

3 Assessing forward guidance puzzles

In this section, we use IE-stability to analyze FGAs in a series of different prominent and commonly used models. We draw comparisons to the analysis done by Carlstrom et al. (2015) and show how IE-stability generalizes their results to offer new insights.

3.1 Models without lagged endogenous variables

We start by studying the standard three equation New Keynesian model:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma\left(i_t - \mathbb{E}_t \pi_{t+1} - r^n\right) \tag{12}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t \tag{13}$$

$$i_t = r^n + \phi_\pi \pi_t + \phi_y y_t, \tag{14}$$

where x is the output gap, π is inflation, and i is the nominal interest rate. The model's reduced form is given by

$$\Gamma = \begin{pmatrix} 0 \\ 0 \\ r^n \end{pmatrix}, \; A = 0_{3\times 3}, \; B = \begin{pmatrix} \frac{1}{\kappa\sigma\phi_\pi+\sigma\phi_y+1} & \frac{\sigma-\beta\sigma\phi_\pi}{\kappa\sigma\phi_\pi+\sigma\phi_y+1} & 0 \\ \frac{\kappa}{\kappa\sigma\phi_\pi+\sigma\phi_y+1} & \frac{\sigma\phi_y\beta+\beta+\kappa\sigma}{\kappa\sigma\phi_\pi+\sigma\phi_y+1} & 0 \\ \frac{\kappa\phi_\pi+\phi_y}{\kappa\sigma\phi_\pi+\sigma\phi_y+1} & \frac{(\beta+\kappa\sigma)\phi_\pi+\sigma\phi_y}{\kappa\sigma\phi_\pi+\sigma\phi_y+1} & 0 \end{pmatrix}, \; D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

The model has no lagged endogenous variables or exogenous shocks so the relevant IE-stability depends are the eigenvalues of the B matrix. The eigenvalues are

$$\lambda_{1,2} = \frac{1+\beta+\kappa\sigma+\beta\sigma\phi_y \pm \sqrt{(\beta+\kappa\sigma+\beta\sigma\phi_y+1)^2 - 4\beta\left(\kappa\sigma\phi_\pi+\sigma\phi_y+1\right)}}{2\left(\kappa\sigma\phi_\pi+\sigma\phi_y+1\right)}$$

and $\lambda_3 = 0$. The eigenvalues are within the unit circle so long as

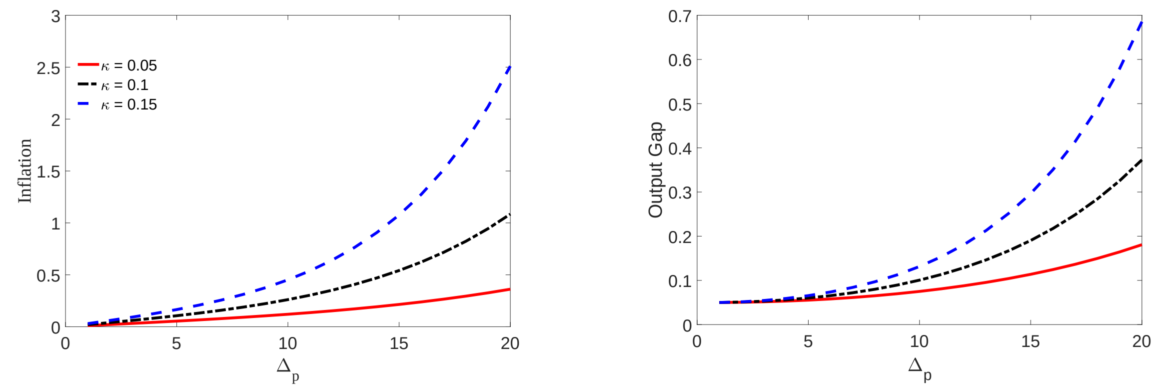
$$\phi_\pi > \frac{\kappa+(\beta-1)\phi_y}{\kappa},$$

which coincides exactly with the condition for determinacy in this model. Clearly, if $\phi_\pi = \phi_y = 0$, then determinacy and IE-stability are not satisfied. However, the condition also reveals that any constellation of parameters that does not satisfy the above condition also provide the necessary environment to generate puzzling behavior for any anticipated change in the economy. Figure 1 provides three examples.

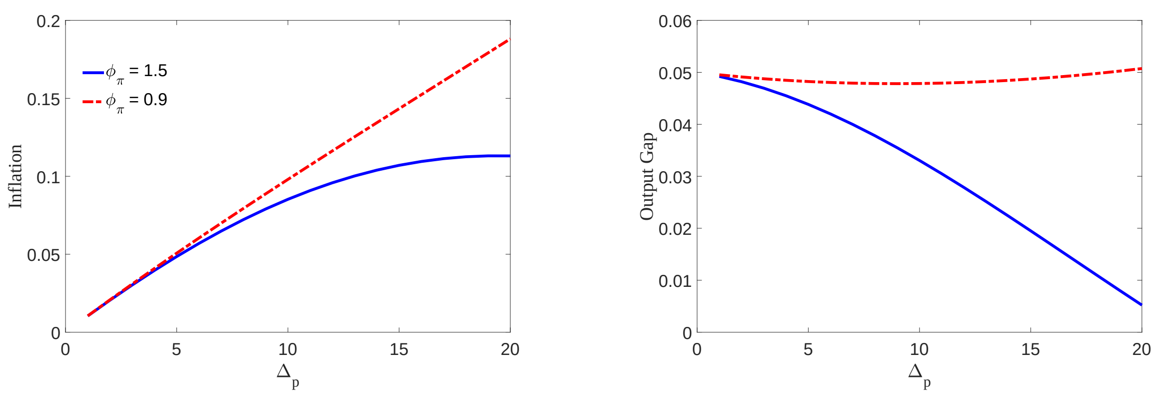
The first set of figures replicates the analysis of Carlstrom et al. (2015) by showing the dependence of the impact of forward guidance on the slope of the Phillips curve. In this forward guidance excercise, the central bank announces a one-time 25 basis point promised reduction of the interest rate Δ_p periods in the future when interest rates are pegged (i.e. $\phi_\pi = \phi_y = 0$) from the time of announcement T^a through implementation T^*, after which time active monetary resumes (e.g. $\phi_\pi > 1$). When the solution is indeterminate, there is an explosive root when the system is solved backwards, which

Figure 1: Initial Response to Forward Guidance Announcements

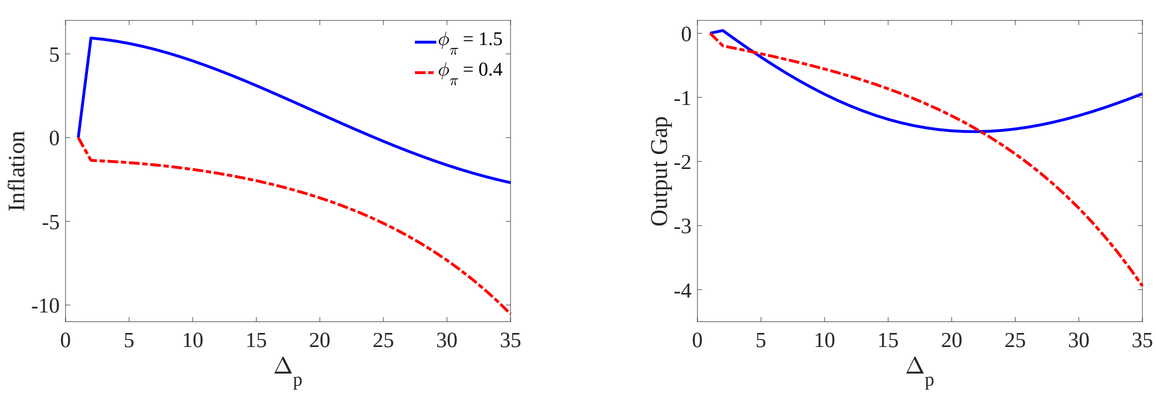
The FGP and the slope of the Phillips curve



The FGP and indeterminacy



The FGP and announced changes to the inflation target



Notes: The figure shows the initial response of output and inflation to forward guidance announcements (FGA).

generates the puzzle. The root Carlstrom et al. (2015) study is

$$\left(1+\frac{\kappa}{2\sigma}\right)+\sqrt{\left(1+\frac{\kappa}{2\sigma}\right)^{2}-1},$$

which is our λ_1 when $\phi_\pi = \phi_y = 0$ and $\beta = 1$. Therefore, IE-stability analysis exactly identifies the same conditions as Carlstrom et al. (2015) and the same intuition about how the slope of the Phillips curve and the intertemporal elasticity of substitution matter for the size of the forward guidance puzzle is obtained. We draw attention to this last point by plotting the impulse responses for different values of κ. As we increase κ, and therefore price flexibility, we increase λ_1 and this exacerbates the impact of forward guidance. Kiley (2016) refers to this heightened sensitivity at higher levels of κ as the "paradox of volatility" and our analysis relates the degree of IE-instability (i.e. the magnitude of λ_1) to this property of the model.

The second set of figures shows how IE-stability generalizes the previous results beyond zero interest rate regimes. These figures compare the impacts of a one-time 25 basis point promised reduction of the interest rate Δ_p periods in the future when policy satisfies the above condition ($\phi_\pi = 1.5$ and $\phi_y = 0$) from the time of announcement T^a through implementation T^* against when policy does not ($\phi_\pi = 0.9$ and $\phi_y = 0$). Despite the absence of an interest rate peg, the latter case still exhibits the Forward Guidance Puzzle.[6]

The third set of figures shows how IE-stability analysis generalizes the Forward Guidance Puzzles to any type of announced event. Here we show the initial response of inflation and the output gap to an announced 2 percentage point increase in the inflation target to take place Δ_p periods in the future when IE-stability of the FGA is satisfied and when it is not. Again, we find unbounded responses when our IE-stability conditions are not satisfied.

[6]Maliar and Taylor (2018) also note that satisfying the Taylor Principle is a sufficient condition for bounded responses in a New Keynesian model without lagged endogenous variables.

Instead of considering a change in the inflation target to illustrate the existence of many forward guidance puzzles, we could redefine θ_{T^*} to model forward fiscal guidance, or the effects of anticipated productivity increases ("paradox of toil"), when interest rates are pegged. But since these exercises ultimately involve the same unique and IE-unstable $\bar{\phi}(\theta_{T^a})$, by the corollary to Proposition 1 we know that all of these exercises will generate a Forward Guidance Puzzle. In other words, the IE-instability of the uniquely determined $\bar{\phi}(\theta_{T^a})$ allows us to diagnose many Forward Guidance Puzzles, including the forward fiscal guidance puzzle of Canzoneri et al. (2018) and the "paradox of toil" in Kiley (2016), in one fell swoop.

3.1.1 Forward guidance puzzle resolutions

There are a number of recent papers that try to resolve the Forward Guidance Puzzle by appealing to some form of bounded rationality or heterogeneity and incomplete markets. Examples of the former are Gabaix (2016) and Angeletos and Lian (2018), while examples of the latter are Eggertsson and Mehrotra (2014), McKay et al. (2016), and Eggertsson et al. (2019). A principle finding among these paper is that bounded rationality or borrowing constraints can dampen agents' responses to announcements about future policy either because they are myopic, believe others are myopic, or because some agents are off of their Euler equations.

The additional discounting and muted responses to future events implied by these models is well-approximated by a linear RE model of the following form

$$
\begin{aligned}
x_t &= M\mathbb{E}_t x_{t+1} - \sigma\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \\
\pi_t &= M^f \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t \\
i_t &= \phi_\pi \pi_t,
\end{aligned}
$$

where M and M^f represent additional discounting of future expectations. IE-stability can be used to quickly assess whether these assumptions can restore IE-stability.

IE-stability implies that the relevant eigenvalues governing the impact of an FGA are

$$\lambda_{1,2} = \frac{M + \beta M^f + \kappa\sigma \pm \sqrt{(\kappa\sigma + M + \beta M^f)^2 - 4\beta M M^f (\kappa\sigma\phi_\pi + 1)}}{2\kappa\sigma\phi_\pi + 2}.$$

Again, we can recover a condition that relates policy and structural parameters to provide IE-stability

$$\phi_\pi + \frac{(1 - \beta M^f)(1 - M)}{\kappa\sigma} > 1.$$

Gabaix (2016) specifically notes an attenuation in the responses of output and inflation to zero interest rate FGA when $\phi_\pi = 0$, $M = .85$, $M^f = .80$, $\kappa = .11$, $\beta = .99$, and $\sigma = .2$. Gabaix's calibration easily satisfies the above condition for IE-stability and therefore will yield bounded results.

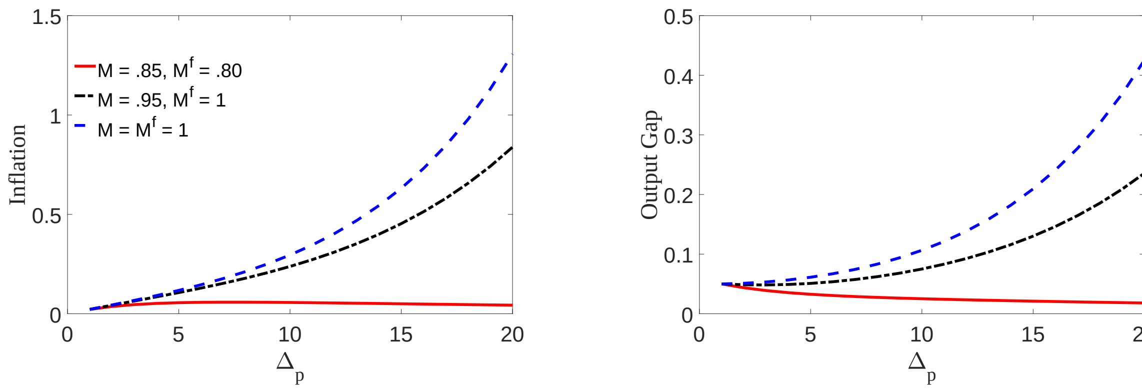
However, not all calibrations will satisfy this condition. Figure 2 shows the impact as Δ_p is increased for different calibration of M and M^f. As these discount rates approach one, the Forward Guidance Puzzle returns. Therefore, whether or not these additional assumptions resolve the Forward Guidance Puzzle depend on how much additional discounting is generated.[7]

3.2 Models with lagged endogenous variables

Proposition 1 makes clear that IE-stability is a locally sufficient condition for bounded responses to FGAs when models include lagged endogenous variables. In these cases, the impact of an FGA is policy specific in that it explicitly depends on the θ_{T^*} regime.

[7] In overlapping generations models, Gibbs (2018) notes that there are two zero interest rate cases in Eggertsson and Mehrotra (2014) with different stability properties under some parameterizations of the model. A zero interest rate equilibrium that is E-stable and one that is not. Therefore, for the same calibration of the model, the impact of forward guidance can also depend on which steady state the dynamics are close to at the time of the announcement.

Figure 2: Forward Guidance Fixes with Added Discounting

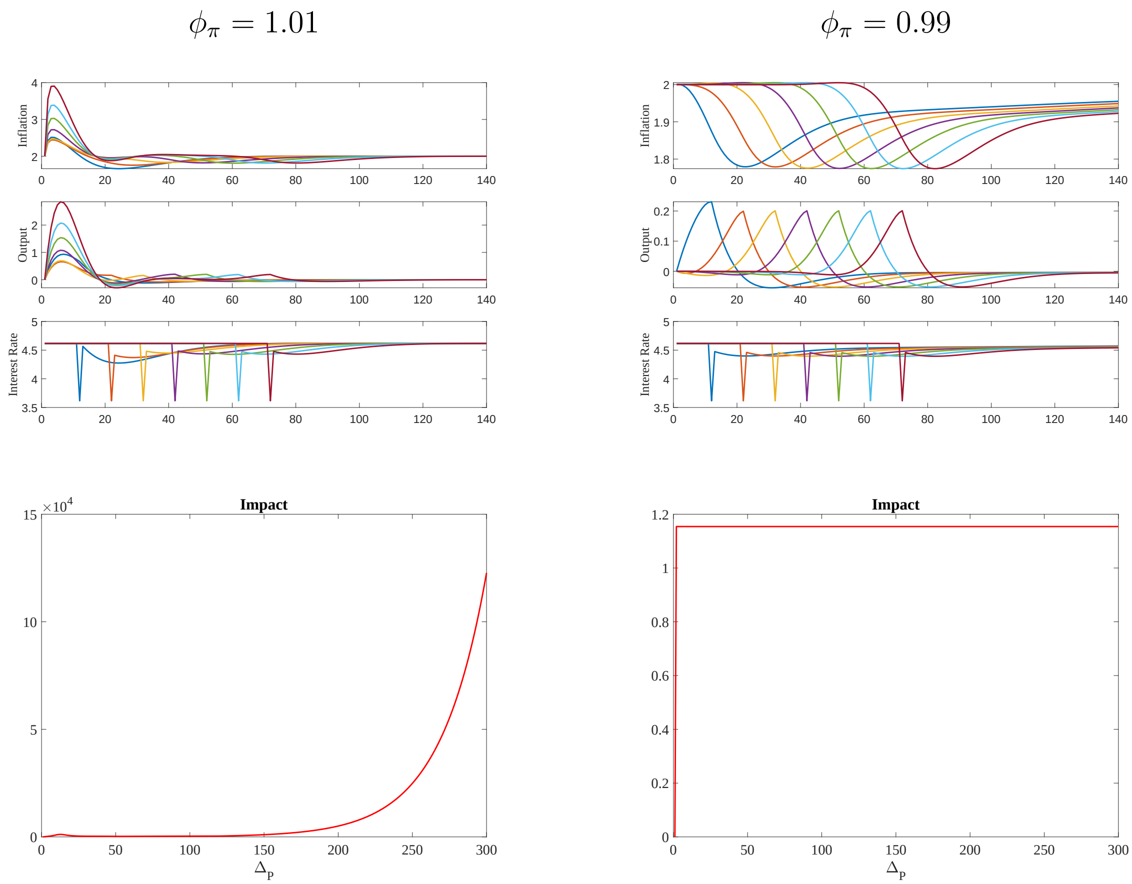


Notes: The initial response of inflation and output in a model that approximates the feature of Gabaix (2016) Angeletos and Lian (2018), Eggertsson and Mehrotra (2014), and McKay et al. (2016).

Carlstrom et al. (2015) touches on this fact in their exploration of the so-called "Reversal Puzzle," where the sign of the initial response to an interest rate FGA can flip as Δ_p is varied in models with inflation indexation. The introduction of an endogenous state variable induces a dependence on the terminal state of economy and can result in complex eigenvalues governing the backward recursion of the solution. The complex eigenvalues cause the impact of an FGA to oscillate as Δ_p is varied leading to the reversals. IE-stability recovers the same eigenvalue conditions as discussed in Carlstrom et al's paper but illuminates the possibility of another outcome, which is perhaps more quantitatively relevant. The terminal announced policy can determine whether any puzzling behavior occurs at all.

We have found a compelling and empirically relevant example. In the model of Smets and Wouters (2007), we can generate both bounded and unbounded responses to zero interest rate policies for the same θ_{T^a} at the models estimated parameters values as predicted by Proposition 1. Specifically, it is possible to construct an IE-stable θ_{T^a} regime at the zero lower bound by introducing a well-defined sunspot following Bianchi and Nicolò (2017). Bianchi and Nicolò shows that you can determine an otherwise indeterminate model by conditioning on an autoregressive sunspots of the following form

Figure 3: Forward Guidance Announcements in the Smets and Wouter's model



Notes: The top figures shows the paths of output, inflation, and interest rates for anticipated 100 basis point monetary policy shocks with $\Delta_p = 10, 20, \ldots, 70$. The bottom figures show the impact using the L_∞ norm for $\Delta_p = 1, \ldots, 300$.

$$s_t = \frac{1}{\alpha} s_{t-1} - \nu_t + \eta_t, \tag{15}$$

where ν_t is the sunspot, η_t is an expectation error from one of the forward looking endogenous variable such as $\pi_t - \mathbb{E}_{t-1}\pi_t$, and α is a free parameter. When $\alpha < 1$, the sunspot process introduces the necessary explosive root to the dynamic system to determine the solution. When $\alpha > 1$, the RE solution does not depend on the sunspot and it has no impact.

Figure 3 shows the dynamic response of output, inflation and the interest rates to forward guidance announcements of an increasing duration for two different FGAs. The

θ_{T^a} regime is the same for both FGAs with a complete suspension of the monetary policy rule and all other parameter set to the estimated mean values reported in Smets and Wouters (2007). For the $t = T^*$ regime, all parameters are held constant except for the monetary policy rule which is reduced to

$$i_t = \phi_\pi \pi_t. \tag{16}$$

In the first FGA, $\phi_\pi = 1.01$ and in the second $\phi_\pi = 0.99$.[8] In the former case, the impact of the policy is increasing in Δ_p. However, when $\phi_\pi < 1$, there is a well-behaved bounded response.

Further numerical exploration suggests that the boundary of the basin of attraction for the sunspot θ_{T^a} regime coincides with standard conditions necessary for determinate monetary policy.[9] Therefore, IE-stability reveals that one explanation for why monetary policy forward guidance may not be arbitrarily powerful is that agents believe that policy will remain passive after lift off and there is coordination on a sunspot. The fact that these impacts can be generated easily at the empirically relevant parameter values suggests that this prediction is plausible. Moreover, it is possible to empirically test this hypothesis using methods similar to those considered in Lubik and Schorfheide (2004) in conjunction with the methods of Kulish and Pagan (2017). This is a compelling case for future research.

Finally, it is worth noting that IE-stability analysis is easy to conduct in models of this size. It requires only a few lines of code to check the relevant eigenvalues from matrices that are typically already constructed for any numerical study of these models.

[8] The sunspot shock is parameterized as $s_t = \frac{1}{\phi_\pi} s_{t-1} - \nu_t + \pi_t - \mathbb{E}_{t-1}\pi_t$ in the simulations, where ϕ_π is the same parameter as in equation (16).

[9] In relation to section 2.3, the assumption of sunspot and passive monetary policy after lift off generates a θ_{T^*} that selects a backward-stable equilibrium. This is because the corresponding $\phi_0(\theta_{T^*})$ is in the basin of attraction of an IE-stable $\bar{\phi}(\theta_{T^a})$.

4 Determinacy, indeterminacy, and IE-stability

The Smets and Wouters example shows how to construct an indeterminate solution that satisfies IE-stability. However, that solution relied on the Bianchi and Nicolò (2017) approach to "determine" a specific sunspot solution. To provide a more familiar example where determinacy, indeterminacy and IE-stability do not overlap, we turn to Markov-switching DSGE models. This also has the added benefit of showing that our IE-stability conditions extend to another popular way to model the zero lower bound.

Following McClung (2019), we generalize our methods to a class of Markov-switching structural models of n equations of the form

$$y_t = \Gamma(\theta, s_t) + A(\theta, s_t) y_{t-1} + B(\theta, s_t)\mathbb{E}_t y_{t+1} + D(\theta, s_t)\omega_t \quad (17)$$

$$\omega_t = \rho(\theta, s_t)\omega_{t-1} + \varepsilon_t \quad (18)$$

where S-state exogenous Markov process, and all variables are defined in section 2. Let P denote the transition probability matrix governing the evolution of s_t and define $p_{ij} = Pr(s_t = j | s_{t-1} = i)$. We assume the model steady state is independent of s_t (e.g. the steady state for $t < T^a$, y_{ss}, does not depend on s_t). This is a common assumption in the literature, particularly in analyses of regime-switching models of monetary-fiscal policy interactions where recurring changes in the monetary and fiscal policy stance do not impact the steady state. A growing literature examines models of the form (17). We will not review the Markov-switching DSGE literature here, but emphasize that Cho (2016, 2019), Maih (2015), Foerster et al. (2016), Farmer et al. (2009, 2011) and Barthélemy and Marx (2019) develop solution techniques for models of this form. Our approach most closely resembles Cho (2016).

The only thing that distinguishes (17) from (1) is the regime-switching variable, s_t, which helps us model recurring changes in the structure of the economy, such as changes in the stance of monetary or fiscal policy. In this class of models, agents do not know the future path of s_t, and therefore s_t allows us to model any uncertainty about the economy's future structure that remains after a FGA. For example, agents may remain uncertain about the future stance of fiscal policy after a monetary forward guidance announcement, and this uncertainty may reduce the impact of a FGA (McClung, 2019). Alternatively, agents could place some probability on a central bank deviating from its FGA in some future period (similar to Haberis et al. (2019)).

Since (17) is so similar to (1), Definitions 1 through 4 extend to models of the form (17). As in Section 2, we obtain the RE solution to a FGA by using the method of undetermined coefficients and backward induction (i.e. our approach is a backward application of techniques developed in Cho (2016)). The method proceeds as follows. The (MSV) RE solution for $t = T^*$ takes the form of

$$y_t = a(s_t) + b(s_t)y_{t-1} + c(s_t)\omega_t \tag{19}$$

This implies that the expectation of y_{t+1} in time T^* is given by

$$\begin{aligned} \mathbb{E}_t y_{t+1} &= \mathbb{E}_t a(s_{t+1}) + \mathbb{E}_t b(s_{t+1})y_t + \mathbb{E}_t c(s_{t+1})\rho(\theta_{T^*}, s_{t+1})\omega_t && (20) \\ &= \sum_{j=0}^{S} p_{s_t j} \left(a(j) + b(j)y_t + c(j)\rho(\theta_{T^*}, j)\omega_t\right) && (21) \end{aligned}$$

Define $b = (b(1), \ldots, b(S))$, $\Xi_*(s_t, b) = (I - B_*(s_t)\mathbb{E}_t b(s_{t+1}))$, and $B_*(s_t) = B(\theta_{T^*}, s_t)$, $B_a(s_t) = B(\theta_{T^a}, s_t)$, etc. Substituting equation (20) into equation (17) and rearranging yields the following equivalences

$$\begin{aligned} a(s_t) &= \Xi_*(s_t, b)^{-1}\left(\Gamma_*(s_t) + B_*(s_t)\mathbb{E}_t a(s_{t+1})\right) && (22) \\ b(s_t) &= \Xi_*(s_t, b)^{-1} A_*(s_t) && (23) \\ c(s_t) &= \Xi_*(s_t, b)^{-1}\left(B_*(s_t)\mathbb{E}_t c(s_{t+1})\rho_*(s_{t+1}) + D_*(s_t)\right) && (24) \end{aligned}$$

where the MSV RE solution is given by $\bar{a}_*(\theta_{T^*}, s_t)$, $\bar{b}_*(\theta_{T^*}, s_t)$, and $\bar{c}(\theta_{T^*}, s_t)$ satisfying equations (22), (23), and (24) for $s_t = 1, \ldots, S$.

In period $t = T^* - 1$, the MSV solution again takes the same form as equation (19). Expectations of y_{t+1} at time $t = T^* - 1$, however, are no longer unknown. They are given by

$$\mathbb{E}_t y_{t+1} \quad = \quad \mathbb{E}_t \bar{a}(\theta_{T^*}, s_{t+1}) + \mathbb{E}_t \bar{b}(\theta_{T^*}, s_{t+1}) y_t + \mathbb{E}_t \bar{c}(\theta_{T^*}, s_{t+1}) \rho_*(s_{t+1}) \omega_t$$

Define $\bar{b}(\theta_{T^*}) = (\bar{b}(\theta_{T^*}, 1), \ldots, \bar{b}(\theta_{T^*}, S))$. Substituting expectations into equation (17) and equating with equation (19), we now have the following equivalences

$$\begin{aligned}
a(s_t) &= \Xi_a(s_t, \bar{b}(\theta_{T^*}))^{-1} \left(\Gamma_a(s_t) + B_a(s_t) \mathbb{E}_t \bar{a}(\theta_{T^*}, s_{t+1})\right) \\
b(s_t) &= \Xi_a(s_t, \bar{b}(\theta_{T^*}))^{-1} A_a(s_t) \\
c(s_t) &= \Xi_a(s_t, \bar{b}(\theta_{T^*}))^{-1} \left(B_a(s_t) \mathbb{E}_t \bar{c}(\theta_{T^*}, s_{t+1}) \rho_a(s_{t+1}) + D_a(s_t)\right)
\end{aligned}$$

which defines the RE solution for $t = T^* - 1$. Continuing to work backwards in time, the entire RE markov-switching solution for the FGA can be written recursively as

$$\bar{a}_j(s_t) = \Xi_a(s_t, \bar{b}_{j-1})^{-1} \left(\Gamma_a(s_t) + B_a(s_t) \mathbb{E}_t \bar{a}_{j-1}(s_{t+1})\right) \tag{25}$$

$$\bar{b}_j(s_t) = \Xi_a(s_t, \bar{b}_{j-1})^{-1} A_a(s_t) \tag{26}$$

$$\bar{c}_j(s_t) = \Xi_a(s_t, \bar{b}_{j-1})^{-1} \left(B_a(s_t) \mathbb{E}_t \bar{c}_{j-1}(s_{t+1}) \rho_a(s_{t+1}) + D_a(s_t)\right) \tag{27}$$

where j is defined as before and $\bar{a}_0 = (\bar{a}(\theta_{T^*}, 1), \ldots, \bar{a}(\theta_{T^*}, S))$, $\bar{b}_0 = (\bar{b}(\theta_{T^*}, 1), \ldots, \bar{b}(\theta_{T^*}, S))$, and $\bar{c}_0 = (\bar{c}(\theta_{T^*}, 1), \ldots, \bar{c}(\theta_{T^*}, S))$.

The T-map difference equations are now (25), (26), and (27). As in Section 2, a fixed point of this map, $\bar{\phi}$, is said to be IE-stable if for all ϕ_0 in an appropriate neighborhood of $\bar{\phi}$, $\phi_N \to \bar{\phi}$ as $N \to \infty$. Likewise, McClung (2019b) provides E-stability conditions, which can be generalized to IE-stability.

Proposition 2 *An MSV solution* $\bar{a} = (\bar{a}(1), \ldots, \bar{a}(S))$*,* $\bar{b} = (\bar{b}(1), \ldots, \bar{b}(S))$*, and* $\bar{c} = (\bar{c}(1), \ldots, \bar{c}(S))$ *is IE-stable if all eigenvalues of*

$$
\begin{aligned}
DT_a(\bar{a}, \bar{b}) &= \left(\oplus_{k=1}^{S} \left(I - B(k) \sum_{h=1}^{S} p_{kh} \bar{b}(h) \right)^{-1} B(k) \right) (P \otimes I_n) \\
DT_b(\bar{b}) &= \left(\oplus_{k=1}^{S} \bar{b}(k)' \otimes \left(I - B(k) \sum_{h=1}^{S} p_{kh} \bar{b}(h) \right)^{-1} B(k) \right) (P \otimes I_n)
\end{aligned}
$$

have modulus less than 1. The solution is not IE-stable if any of the eigenvalues have modulus larger than 1.[10]

The IE-stability condition in Proposition 2 is the local stability condition for a given solution to (25), (26), and (27).[11] If we set $S = 1$, the model (17) becomes (1) and the IE-stability conditions in Proposition 2 become the IE-stability conditions in Theorem 1. Because Definitions 1 through 4 extend to (17), and because y_{ss} does not depend on s_t, we can extend Proposition 1 to determine when a model of the form (17) predicts a Forward Guidance Puzzle.[12]

McClung (2019b) also shows that the IE-stability conditions for solutions to (17) are considerably weaker than the necessary and sufficient conditions for determinacy derived in Cho (2016, 2019). This means that it easy for us to use (17) to demonstrate that it is not indeterminacy that makes a model susceptible to a Forward Guidance Puzzle, as Carlstrom et al. (2015) suggests.

To illustrate, we use a New Keynesian model with time-varying fiscal and monetary policy regimes. The equilibrium conditions for the model include (12) and (13), as in the simple New Keynesian model, but the model also includes a fiscal authority that

[10]The sum operator, $\oplus$, is defined such that $\oplus_{k=1}^{S} A(k) = diag(A(1), \ldots, A(S))$ for generic $n \times n$ matrices $(A(1), \ldots, A(S))$.

[11]We omit the IE-stability condition associated to $DT_c(\bar{b}, \bar{c})$ for exposition's sake, but McClung (2019b) shows that if the eigenvalues of $DT_a(\bar{a}, \bar{c})$ and $DT_b(\bar{b})$ are less than one in magnitude then so are the eigenvalues of $DT_c(\bar{b}, \bar{c})$.

[12]The proof of Proposition 1 in the appendix includes this case.

nominal fiscal surpluses, T, and issues nominal debt, B, to finance fiscal deficits (i.e. when $T < 0$). If we use b_t (τ_t) to denote the log deviation of the real-value level, then we can completely describe the fiscal authority's behavior according to the following policy for τ and intertemporal budget constraint:

$$b_t = \beta^{-1}(b_{t-1} - \pi_t) + i_t - \tau_t \tag{28}$$

$$\tau_t = \gamma(s_t) b_{t-1} \tag{29}$$

The value of γ in (29) is referred to as the fiscal stance on debt. One may better understand the influence of γ on debt-dynamics by substituting (29) into (28), which yields an autoregressive process for b_t with AR(1) parameter equal to $\beta^{-1} - \gamma$. When γ is high (i.e. values of γ consistent with $|\beta^{-1} - \gamma| < 1$), increases in government debt, b, are fully amortized by increases in current and future fiscal surpluses, and b_t evolves according to a stable autoregressive process. On the other hand, low values of γ (i.e. $|\beta^{-1} - \gamma| > 1$) lead to unstable debt-dynamics, and it immediately follows that changes in inflation are required to stabilize debt in equilibrium. We assume that the fiscal stance, γ, varies over time (e.g. as new Congresses pursue different fiscal policies), and that said time-variation in γ is given by an exogenous 2-state Markov process, $s_t \in \{M, F\}$, such that $|\beta^{-1} - \gamma(M)| < 1 < |\beta^{-1} - \gamma(F)|$.[13]

The monetary regime is characterized by a time-varying Taylor rule of the form:

$$i_t = \phi_\pi(s_t)\pi_t \tag{30}$$

where the restriction $\phi_\pi(F) < 1 < \phi_\pi(M)$ is imposed by θ_{T^*}. We impose this last parameter restriction because passive monetary policy (i.e. $\phi_\pi(F) < 1$) allows for debt-stabilizing inflation to occur when $1 < |\beta^{-1} - \gamma(F)|$, whereas active monetary policy

[13] Closely-related models are studied by Davig and Leeper (2011), Bianchi and Ilut (2017), and Bianchi and Melosi (2017), among others.

(i.e. $1 < \phi_\pi(M)$) helps to prevent coordination on sunspots during periods where debt is being stabilized by fiscal policy (i.e. $|\beta^{-1} - \gamma(F)| < 1$).[14]

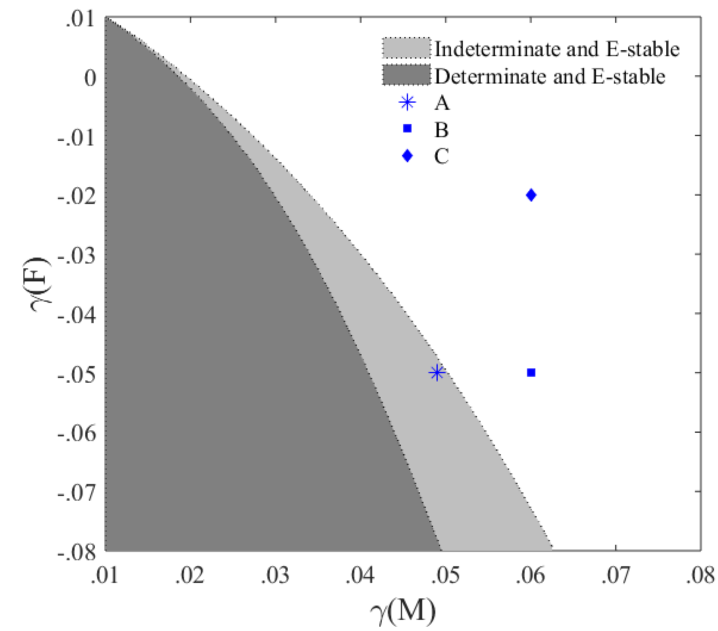
Unlike in the fixed policy regime models examined previously, we cannot analytically describe determinacy in models with recurring regime changes. To assess the determinacy and IE-stability properties of our regime-switching New Keynesian model under an interest rate peg, we use numerical techniques and the conditions derived in Proposition 2. Figure 4 shows determinacy and IE-stability regions in the fiscal policy parameter space under an interest rate peg. Recall that a central theme in this paper is that IE-instability under a peg predicts a Forward Guidance Puzzle. To show that the Forward Guidance Puzzle is independent of indeterminacy, we examine three parameterizations from Figure 4. Parameterization A delivers indeterminacy under an interest rate peg, but the corresponding MSV solution is IE-stable. Parameterization B constitutes a small deviation from Parameterization A, yet this minor deviation is enough to render the model IE-unstable under the peg. Parameterization C is still further in the indeterminacy, IE-unstable region of the policy parameter space.[15]

According to our IE-stability criterion, Parameterization A should correspond to a Puzzle-proof model, whereas Parameterizations B and C should not. We show that our main result holds here. Figure 5 plots the initial responses of inflation and output to anticipated policy shocks for increasing Δ_p's for the three parameterizations. In these plots we assume that the economy is in Regime M at the time of the forward guidance announcement, though qualitatively similar impulse responses can be obtained under the alternative assumption that the economy is in Regime F.

[14] McClung (2019) uses a similar model to show when the forward guidance puzzle is ameliorated by fiscal policy considerations. Since our only goal in this section is to write down a full model (given by (12)-(13) and (28)-(30)) that allows us to assess the relationship between indeterminacy, IE-instability and the forward guidance puzzle, we refer interested readers to McClung (2019) for more information on the model itself.

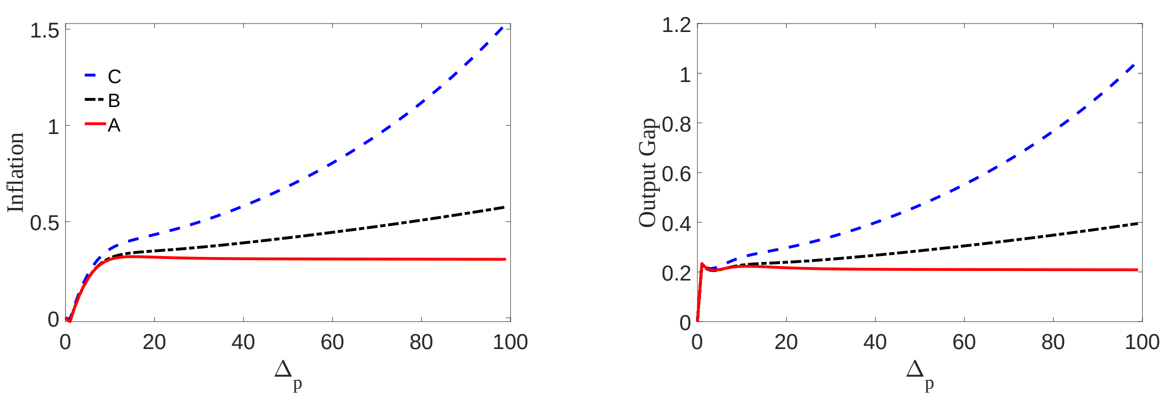
[15] We can construct stable common-factor sunspot solutions for each of three parameterizations considered in this section. Hence, multiple equilibria exist under Parameterizations A, B, C. See Cho (2016, 2019) for more information.

Figure 4: IE-stability and Determinacy Regions



Notes: The white region is the indeterminate and IE-unstable parameter region.

Figure 5: Forward Guidance Puzzles, Indeterminacy and E-stability



Notes: The initial responses of inflation and output in the Markov-switching New Keynesian model.

5 Variations on a theme

Kiley (2016) and Carlstrom et al. (2015) have proposed resolutions to the Forward Guidance Puzzle that appeal to the sticky-information New Keynesian framework of Mankiw and Reis (2002). Sticky-information models have a time-varying structure that prevent us from directly applying our IE-stability criterion. Nonetheless, we can employ some basic intuition from our framework to rationalize their results.

In the basic sticky information model, the price level, p, and the output gap, x, evolve according to the following IS equation and sticky-information Phillips equation:

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - (\mathbb{E}_t p_{t+1} - p_t)) \tag{31}$$

$$p_t = \lambda \sum_{k=0}^{\infty} (1-\lambda)^k \, \mathbb{E}_{t-k}(p_t + \alpha x_t) \tag{32}$$

Consider a standard zero interest rate FGA in this model: at $t = T^a$ the central bank announces a a suspension of active monetary policy from $t = T^a$ to $t = T^* - 1$ (i.e. $i_t = 0$ for $T^a \leq t \leq T^* - 1$), a one-time cut in the interest rate at $t = T^*$ (i.e. $i_{T^*} = \bar{i} < 0$), and resumption of the Taylor rule with active monetary policy for all $t > T^*$. Because expectations based on pre-announcement information equal zero (e.g. $\mathbb{E}_{t-k}(p_t + \alpha x_t) = 0$ for $k > t - T^a$) we can rewrite (32) as:

$$p_t = \lambda \sum_{k=0}^{t-T^a} (1-\lambda)^k \, \mathbb{E}_{t-k}(p_t + \alpha x_t)$$

Moreover, since all information is revealed at $t = T^a$, $\mathbb{E}_{t-k}(p_t + \alpha x_t) = p_t + \alpha x_t$ for all $k = 0, \ldots, t - T^a$, and (32) further reduces to

$$\begin{aligned} p_t &= a(t - T^a) x_t \\ a(t - T^a) &= \frac{1 - (1-\lambda)^{t+1-T^a}}{(1-\lambda)^{t+1-T^a}} \alpha \end{aligned} \tag{33}$$

We can substitute for x_t and $\mathbb{E}_t x_{t+1}$ in (31) using (33) to derive the equilibrium law of motion for p_t:

$$
\begin{aligned}
p_t &= \frac{\sigma + a(t+1-T^a)^{-1}}{\sigma + a(t-T^a)^{-1}}\mathbb{E}_t p_{t+1} - \frac{\sigma}{\sigma + a(t-T^a)^{-1}} i_t \\
&= a_p(t-T^a)\mathbb{E}_t p_{t+1} - \sigma_p(t-T^a) i_t.
\end{aligned}
$$

Since the Taylor rule is satisfied for $t > T^*$ we furthermore know that $p_t = x_t = 0$ for all $t > T^*$. Hence, it follows from the design of the experiment that

$$
\begin{aligned}
p_{T^*} &= \sigma_p(T^* - T^a)\bar{i} \\
p_t &= a_p(t-T^a)\mathbb{E}_t p_{t+1} \ \forall T^a \leq t \leq T^* - 1.
\end{aligned}
$$

Since $a(t-T^a)$ is increasing in t, $0 < a_p(t-T^a) < 1$ for $t < \infty$, and $\lim_{t\to\infty} a_p(t-T^a) = \lim_{j\to\infty} \sigma_p(t-T^a) = 1$. Consequently, p_t rises from p_{T^a} to p_{T^*}, then falls abruptly at the end of the forward guidance period. Since prices do not explode backwards, the sticky information model does not exhibit the Forward Guidance Puzzle. Prices do not explode precisely because $|a_p(t-T^a)| < 1$. Though $a_p(t-T^a)$ is time-varying, it is bounded above by some $\tilde{a} < 1$ for values of t sufficiently close to T^a. If we study the approximate first-order system for prices formed by replacing $a_p(t-T^a)$ with $\tilde{a}$, the unique solution is IE-stable and satisfies the conditions in Proposition 1.

It is worth noting that while p_{T^a} is non-increasing in T^*, y_{T^a} is non-decreasing in T^*. To see this, substitute for p_t and $\mathbb{E}_t p_{t+1}$ in (31) using (33):

$$
\begin{aligned}
x_t &= \frac{\sigma a(t+1-T^a)+1}{\sigma a(t-T^a)+1}\mathbb{E}_t x_{t+1} - \frac{\sigma}{\sigma a(t-T^a)+1} i_t \\
&= a_x(t-T^a)\mathbb{E}_t x_{t+1} - \sigma_y(t-T^a) i_t. \qquad (34)
\end{aligned}
$$

Since $a_x(t-T^a) \geq 1$, x explodes backwards from T^*, seemingly in violation of our

IE-stability result. This result is deceptive. The fact that $a(t - T^a)$ increases in t means that the magnitude of the response of x_{T^*} to i_{T^*} is strictly decreasing in T^* and equal to 0 in the limit as T^* goes to infinity. This creates an upper bound on x_{T^a}, which can be shown formally if we iterate backwards on (34) and solve for x_{T^a}:

$$x_{T^a} = -\sigma \frac{(1-\lambda)}{(1-\lambda) + \lambda\sigma\alpha} \bar{i}$$

which can also be recovered from equation (14) in Kiley (2016).

6 Conclusion

Viewing the many puzzles of forward guidance announcements through the lens of IE-stability reduces the problem to one which we know a great deal about. E-stability and related concepts have been studied for over thirty years now and there exists a number of off-the-shelf results that can quickly be applied to assess, diagnose, and select RE forward guidance predictions in a variety of models. Here, we have shown only a few of the many potential applications.

Appendix

Proof of Proposition 1

Case 1: models of the form of Equation (1) *We can write impact of an FGA* $\{\theta_i\}_{i=T^a,T^*}$ *using equations (9), (10), and (11) as*

$$\begin{aligned} |y_{ss} - \mathbb{E}[y_{T^a}]| &= |y_{ss} - \mathbb{E}\left[(I - B_a\bar{b}_j)^{-1}(\Gamma_a + B_a\bar{a}_j + (B_a\bar{c}_j\rho_a + D_a)\omega_{T^a} + A_a y_{T^a-1})\right]| \\ &= |y_{ss} - (I - B_a\bar{b}_j)^{-1}(\Gamma_a + B_a\bar{a}_j + A_a y_{ss})| \\ &= |(I - B_a\bar{b}_j)^{-1}\left((I - B_a\bar{b}_j - A_a)y_{ss} - (\Gamma_a + B_a\bar{a}_j)\right)|, \end{aligned}$$

where $B_a = B(\theta_{T^a})$ and $A_a = A(\theta_{T^a})$, etc, and $|\cdot|$ is any p-norm. By the triangle inequality, it follows that

$$\begin{aligned}
|(I - B_a\bar{b}_j)^{-1}(I - B_a\bar{b}_j - A_a)y_{ss}| &+ |(I - B_a\bar{b}_j)^{-1}(\Gamma_a + B_a\bar{a}_j)| \\
&\geq |(I - B_a\bar{b}_j)^{-1}\left((I - B_a\bar{b}_j - A_a)y_{ss} - (\Gamma_a + B_a\bar{a}_j)\right)|.
\end{aligned}$$

If all three conditions are satisfied, then $\bar{a}_j \to \bar{a}(\theta_{T^a})$ and $\bar{b}_j \to \bar{b}(\theta_{T^a})$ as $j = \Delta_p \to \infty$ by Theorem 1. Therefore, in the limit as Δ_p goes to infinity the impact of the FGA can be no larger than $|(I - \bar{b}(\theta_{T^a}))y_{ss}| + |\bar{a}(\theta_{T^a})|$. $\square$

Case 2: models of the form of (17) *Recall that y_{ss}, the steady state of the model when $t < T^a$, does not depend on s_t. Therefore, we can write the impact the same as in Case 1:*

$$\begin{aligned}
|y_{ss} &- \mathbb{E}\left[y_{T^a}\right]| = |y_{ss} - \mathbb{E}\left[\Xi_a(s_{T^a}, \bar{b}_j)^{-1}\left(\Gamma_a(s_{T^a}) + B_a(s_{T^a})\sum_{k=1}^{S} p_{s_{T^a}k}\bar{a}_j(k)\right)\right] \\
&- \mathbb{E}\left[\Xi_a(s_{T^a}, \bar{b}_j)^{-1}\left(B_a(s_{T^a})\sum_{k=1}^{S} p_{s_{T^a}k}\bar{c}_j(k)\rho_a(k) + D_a(s_{T^a})\right)\omega_{T^a}\right] \\
&- \mathbb{E}\left[\Xi_a(s_{T^a}, \bar{b}_j)^{-1}A_a(s_{T^a})y_{T^a-1}\right]| \\
&= |y_{ss} - \sum_{i=1}^{S}\bar{\pi}_i\left(\Xi_a(i, \bar{b}_j)^{-1}\left(\Gamma_a(i) + B_a(i)\sum_{k=1}^{S} p_{ik}\bar{a}_j(k) + A_a(i)y_{ss}\right)\right)| \\
&= |\sum_{i=1}^{S}\bar{\pi}_i\left(\Xi_a(i, \bar{b}_j)^{-1}\left(\Xi_a(i, \bar{b}_j) - A_a(i)\right)y_{ss}\right) \\
&- \sum_{i=1}^{S}\bar{\pi}_i\left(\Xi_a(i, \bar{b}_j)^{-1}\left(\Gamma_a(i) + B_a(i)\sum_{k=1}^{S} p_{ik}\bar{a}_j(k)\right)\right)|
\end{aligned}$$

where $\Xi_a(s_{T^a}, \bar{b}_j) = (I - B_a(s_{T^a})\mathbb{E}_{T^a}b(s_{T^a+1}))$, and $\bar{\pi}_i$ is the marginal density of Markov state, i. Then, given that $\bar{a}(\theta_{T^a}) = (\bar{a}(\theta_{T^a}, 1), \ldots, \bar{a}(\theta_{T^a}, S))$ and $\bar{b}(\theta_{T^a}) = (\bar{b}(\theta_{T^a}, 1), \ldots, \bar{b}(\theta_{T^a}, S))$

exist and the IE-stability conditions given by Proposition 2 are satisfied, we can construct the same bound for the impact as $j \to \infty$ as in case 1:

$$
\begin{aligned}
&|\sum_{i=1}^{S} \bar{\pi}_i \left(\Xi_a(i, \bar{b}(\theta_{T^a}))^{-1} \left(\Xi_a(i, \bar{b}(\theta_{T^a})) - A_a(i)\right) y_{ss}\right)| \\
+&|\sum_{i=1}^{S} \bar{\pi}_i \left(\Xi_a(i, \bar{b}(\theta_{T^a}))^{-1} \left(\Gamma_a(i) + B_a(i) \sum_{k=1}^{S} p_{ik} \bar{a}(\theta_{T^a}, k)\right)\right)|
\end{aligned}
$$

$\square$

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