# Online Appendix

## A. Model Details

The modeling environment is a multi-period overlapping generations model in which households are subject to idiosyncratic earnings risk. A household is uniquely identified by the family dynasty they are born into ($i \in \{1, \ldots, N\}$), their age ($j \in \{1, \ldots, J\}$), and the time period, $t$, into which they are born. Thus, a household of dynasty $i$ born in time $t$ is of age $j = 1$ in period $t$, lives for $J$ periods and upon death is replaced by a new household of dynasty $i$ in period $t + J + 1$. As such, the population of the economy at time $t$ is given by $N_t = NJ$.

An agent is tasked with choosing a sequence of savings allocations, $a^{j,i}_{t+j-1}$, for $j = 1, \ldots, J$ with terminal assets $a^{J,i}_{t+J-1} = 0$, as our agents have no bequest motive and consume all available resources in the final period of life. An agent's lifespan is non-stochastic and known to each agent at the beginning of their life. Agents inelastically supply their labor in the first $j_R < J$ periods of their life, however their labor productivity is subject to an idiosyncratic shock that each household must forecast. The asset allocations above are selected to solve the household's optimization problem in each period, outlined below for an arbitrary agent of dynasty $i$ born at time $t$.

$$\max_{\{a^{j,i}_{t+j-1}\}_{j=1}^{J-1}} \hat{E}_t \sum_{j=1}^{J} \beta^{j-1} u(c^{j,i}_{t+j-1}) \tag{1}$$

$$\text{s.t.} \qquad c^{j,i}_{t+j-1} + a^{j,i}_{t+j-1} \leq R_{t+j-1} a^{j-1,i}_{t+j-2} + \epsilon(s^{j,i}_{t+j-1}) h(j) w_{t+j-1} \tag{2}$$

Where $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, $\hat{E}_t$ denotes (potentially) non-rational expectations formed at $t$, $R_{t+j-1}$ and $w_{t+j-1}$ are the economy wide return on savings and labor, respectively, and $s^{j,i}_{t+j-1}$ is a two-state persistent exogenous Markov process governing the idiosyncratic employment risk faced by optimizing households. The transition out of state $s \in \{L, H\}$ s.t. $0 \leq \epsilon(L) < \epsilon(H) = 1$ is governed by the Markov transition probabilities $P_L$ and $P_H$. The high employment state, $\epsilon(H)$, corresponds to full time employment and the low employment state, $\epsilon(L)$, corresponds to agents being "unemployed." We assume that agents observe their employment process and knows the value of $\epsilon(H)$ and $\epsilon(L)$. $h(j)$ is an age-earnings profile. Appendix B describes the calibration of the age-earnings profile.

Furthermore, we assume that agents are born with no wealth (i.e. $a^{0,i}_{t-1} = 0$ for all $t$ and $i$) and that all agents have the same CRRA utility function, given by $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ if $\sigma \neq 1$ and $u(c) = \ln(c)$ otherwise. If we let $x = (a, \epsilon, j)$ then households' optimization problem can be written as a dynamic programming problem:

$$\begin{aligned} V(x) &= \max_{c,a'} \left\{ u(c) + \beta \hat{E} V(x'|x) \right\} \\ \text{s.t. } c + a' &\leq Ra + \epsilon w \end{aligned} \tag{3}$$

The policy functions for savings, $a(x)$, and consumption, $c(x)$, determine the allocations of savings that solve the household's optimization problem, which we recast as (3). From (3), it's immediate that agents' decisions depend on expectations of employment, which may be non-rational, as denoted by $\hat{E}$ . In our analysis, we assume that agents form naive expectations of the real interest rate and wage (i.e $R^e_{t+j} = R_t$ and $w^e_{t+j} = w_t$) since this is consistent with rational expectations of prices in a stationary equilibrium of the model. We furthermore assume that agents understand that personal income is driven by a process of the form (2).

We depart from rational expectations by assuming that a proportion of agents lack knowledge of the true parameters $P_H$ and $P_L$ and instead solve their dynamic programming problems in each period by conditioning expectations on non-rational beliefs, $P^{e,j,i}_{H,t+j-1}$ and $P^{e,j,i}_{L,t+j-1}$. This implies $\hat{E}V(x'|x) = \sum_{\epsilon'} Pr^{e,j,i}_{t+j-1}(\epsilon'|\epsilon) V(a', \epsilon', j+1|x)$ depends on the household's subjective transition probabilities, $Pr^{e,j,i}_{t+j-1}(\epsilon'|\epsilon)$, which may vary across agents and over the course of each agent's lifetime. In subsequent projects, non-rational agents will be tasked with learning other features of their employment process (e.g. a realistically calibrated economy wide age earnings profile, the dispersion of earnings over the life-cycle, etc). However, we chose to model a two-state earnings process with just two transition probabilities for agents to learn in order to display the power of belief diffusion in a simple modeling environment. To make things tractable, we assume that agents solve their optimization problem in each period under the (potentially false) belief that their current beliefs about transition probabilities will not change in future periods (i.e. Kreps' anticipated utility approach). Finally, to pin down each agent's dynamic programming problem, we need to specify the evolution of $P^{e,j,i}_{H,t+j-1}$ and $P^{e,j,i}_{L,t+j-1}$.

**Assumption 1** *A proportion of each generation, $\phi$, have true beliefs about the idiosyncratic income transition probabilities*

$$P^{e,j,i}_{k,t+j-1} = P_k$$

*for $k = H, L$, for all $t$, and for all $i, j$ in the $\phi$ proportion of households. Households belonging to this $\phi$ proportion are called* ***informed (I) households****.*

Informed households always have true beliefs about the persistence of employment and unemployment states. How might a household come to form Informed beliefs? To illustrate one potential explanation, suppose that an adaptive learning agent estimates $P_H$ and $P_L$ using an infinite history of individuals' employment states. Since the employment state process is exogenous, this agent will surely learn the transition probabilities if they use, for example, the following recursive estimator: $P_{k,t} = t_k^{-1}\xi_t + (1 - t_k^{-1})P_{k,t-1}$ if $s_{t-1} = k$ where $t_k \leq t$ is the number of periods that state $k = H, L$ has been realized up until $t - 1$ and $\xi_t = 1$ if $s_t = k$ and 0 if $s_t \neq k$. When $s_{t-1} \neq k$ the recursive estimator is $P_{k,t} = P_{k,t-1}$. The learnability of the moments of exogenous processes is so taken for granted in the adaptive learning literature that most applications simply endow agents with complete information about exogenous processes. Generally, these applications involve infinitely lived agents who are learning about aggregate disturbances to the economy.

In our case, the learning agent would need to collect a lot of personal employment data from multiple finitely lived agents to learn these transition probabilities under general assumptions. How might an agent obtain this information? For one, a public institution could collect and publish these data, but it would be unreasonable to assume that all agents can access this data or would choose to utilize it once given access. Alternatively, we could assume that information is made available within dynasties, $i$, (e.g. by generations who pass down personal income data to the next generation). This is why we refer to households with always true beliefs about transition probabilities, $p_H$ and $p_L$, as Informed households.

**Assumption 2** *A proportion of each generation, $1-\phi$, form beliefs according to*

$$\begin{aligned} P^{e,j,i}_{k,t+j-1} &= \xi^{i,j}_{k,t-j-1}\left(\xi^{i,j-1}_{k,t-j-2}(\gamma_k - \gamma_k P^{e,j-1,i}_{k,t+j-2}) + P^{e,j-1,i}_{k,t+j-2}\right) \\ &+ \left(1-\xi^{i,j}_{k,t-j-1}\right)\left(1-\gamma_k \xi^{i,j-1}_{k,t-j-2}\right) P^{e,j-1,i}_{k,t+j-2} \end{aligned} \tag{4}$$

*given $P^{e,1,i}_{k,t}$ for all $i, j > 2$ in the $1-\phi$ proportion of households, where $0 < \gamma_k < 1$, $\xi^{i,j}_{k,t-j-1} = 1$ if $s^{j,i}_{t-j-1} = k$ and 0 otherwise, and $k = H, L$. Households belonging to this $1-\phi$ proportion are called* ***uninformed households****.*

Two key features of uninformed expectation formation distinguish uninformed households from informed households. First, uninformed households do not know the true transition probabilities, but try to learn these payoff-relevant parameters using personal employment data (i.e. $s^{j,i}_{t+j-1}$) and the learning algorithm (4). Notice that (4) becomes the above mentioned recursive estimator of transition probabilities if we set $\gamma_k$ equal to $1/t_k$ where $k$ is the number of periods agent $(i, j)$ experienced state $k$. Hence (4) can be viewed as a constant gain learning algorithm with gain parameter, $\gamma_k$, that delivers a weighted average of the transition probabilities.

Second, $P^{e,1,i}_{k,t} \neq P_k$ and $P^{e,j,i}_{k,t+j-1} \neq P_k$ are both possible under uninformed learning. This means that agents can overestimate or underestimate their expected future income stream over the life-cycle based on their initial beliefs, $P^{e,1,i}_{k,t}$, or based on personal experiences over the life-cycle. How are these initial beliefs assigned and how do they interact with the learning process?

**Definition 1** *An individual $(i, j)$ born at $t$ is said to be an **optimist (O)** or optimistic if $P^{e,1,i}_{H,t} \equiv P^{O}_{H} \geq P_H$ and $P^{e,1,i}_{L,t} \equiv P^{O}_{L} \leq P_L$, with strict inequality holding for at least one.*

An optimist overestimates the probability of being in the high employment state relative to the unemployment state. Since agents' expectation of future income depends positively (negatively) on $P^{e,j,i}_{H,t+j-1}$ ($P^{e,j,i}_{L,t+j-1}$) through (4), an optimist will overestimate their expected future income stream early in life relative to informed or pessimistic uninformed households.

**Definition 2** *An individual $(i, j)$ born at $t$ is said to be a **pessimist (P)** or pessimistic if $P^{e,1,i}_{H,t} \equiv P^{P}_{H} \leq P_H$ and/or $P^{e,1,i}_{L,t} \equiv P^{P}_{L} \geq P_L$, with strict inequality holding for at least one.*

A pessimist underestimates the probability of being in the high employment state relative to the unemployment state. Consequently, a pessimist will underestimate their expected future income stream early in life relative to informed or optimistic uninformed households.

Our specification of heterogeneous beliefs implies a rich distribution over agent's types. As in other simple models, an agent's type is pinned down by their age, $j$, their asset holdings, $a$, and their employment status, $\epsilon$, but agents are also distinguished by their beliefs, $P^{e,j,i} = (P^{e,j,i}_{H,t+j-1}, P^{e,j,i}_{L,t+j-1})$ which are entirely determined by the agent's initial beliefs, their age, and their employment histories. Importantly, the distribution of beliefs in the economy is time-invariant (stationary) because the distributions over initial beliefs, employment, and the economy's age structure is also time-invariant. This allows us to write the time $t$ distribution over agents' types as $\Lambda(x, P^{e,j,i})$.

To close the model, we assume a standard aggregate production function: $Y_t = K^{\alpha}_t H^{1-\alpha}_t$, where $H_t$ is the effective labor supplied to the market in period $t$, $K_t$ is the aggregate capital stock at $t$, and $Y_t$ is aggregate output. Factor prices are determined by profit maximization in perfectly competitive markets such that

$$
\begin{aligned}
R_t &= \alpha K_t^{\alpha-1} H_t^{1-\alpha} + 1 - \delta && (5)\\
w_t &= (1-\alpha) K_t^{\alpha} H_t^{-\alpha}. && (6)
\end{aligned}
$$

where $\delta$ is the rate of capital depreciation. The economy consists of three markets that need to clear in each period. First, the labor market clears if and only if the number of effective hours worked, $H_t$, equals the number of labor inputs in the economy at time $t$:

$$H_t = \sum_{j=1}^{J} \sum_{i=1}^{N} \epsilon(s_t^{j,i}) \tag{7}$$

Second, the asset market clears if and only if the capital stock, $K_{t+1}$, equals the sum of household savings at $t$:

$$K_{t+1} = \sum_{j=1}^{J} \sum_{i=1}^{N} a_t^{j,i} \tag{8}$$

Finally, the goods market clears by Walras' Law.

A stationary, competitive equilibrium is a collection of aggregate quantities $(K, H, Y)$, prices $(r, w)$, continuation values $(V(x))$, policy functions $(a(x),\ c(x))$, and a distribution of agents' types $(\Lambda(x, P^e))$ such that: 1. policy functions and value continuation functions solve the household's optimization problem; 2. firms maximize profits; 3. the distribution over household types, $\Lambda(x, P^{e,j,i})$, is stationary; 4. prices are given by (4) and (5); 5. markets clear.

## B. Outline of Equilibrium Solution Algorithm

1. Propose a candidate steady state capital stock, $K_0$, and corresponding interest rate, $R$, and wage, $w$, that solve the firm optimization problem.

2. Solve the household problem ($c(x)$ and $a(x)$) given prices $R$ and $w$.

   We use a standard value function iteration approach to solve the household optimization problem. Our value function iteration approach proposes a fine grid over savings allocations, $a$, and a fine grid over transition probability beliefs, $P_H^e$ and $P_L^e$. In each period of each uninformed agent's life, beliefs evolve according to (4) and are then rounded to nearest transition probability pair $(P_L^e, P_H^e)$ in the transition probability grid.

3. Using the optimized capital decision rule, $a(x)$, compute individual savings decisions for the stable distribution of households.

4. Compute aggregate capital, $K_1$, the capital stock in the model economy given preferences. If $K_1$ is sufficiently close to $K_0$, stop. Otherwise, set $K_0 = K_1$ and repeat steps (1)-(4) until convergence is achieved.