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This note is a correction of "The Power of Forward Guidance and the Fiscal Theory of the Price Level" published in the December 2021 IJCB. The posting of this note was agreed upon by the IJCB Editorial Office.

I personally discovered an error in my paper, "The Power of Forward Guidance and the Fiscal Theory of the Price Level", which appeared in the December 2021 issue of the *International Journal of Central Banking*. That paper examines assumptions about monetary-fiscal regimes that rule out the possibility that anticipated shocks to the path of interest rates have unbounded effects as the horizon of the shock is pushed into the infinite future ("New Keynesian forward guidance puzzle"). To that end, section 3 of the paper presents a generalized definition of "forward guidance puzzle", and a general methodology for determining whether a given model is susceptible to the puzzle. Proposition 2 specifically presents "necessary and sufficient" conditions for ruling out the forward guidance puzzle, which can be applied in a broad class of models. Those conditions are necessary and sufficient when applied to the canonical interest rate forward guidance puzzle exercises I study in my paper. That is, the canonical interest rate forward guidance puzzle emerges (is absent) in my applications if the conditions fail (are satisfied). However, for some types of policy announcements that I did not study in the paper, and which are not of particular interest in the broader forward guidance puzzle literature, those conditions are only sufficient for ruling out the puzzle as defined in section 3 of the paper (in fact, the necessary conditions appear in the appendix of the paper). This corrigendum presents the corrected version of Proposition 2.[1]

Proposition 2 (corrected). *A model of the form (11) does not exhibit a forward guidance puzzle if:*

1. $\bar{\Omega}(s_0) = lim_{T_1\to\infty}\Omega_0(s_0)$ *exists for all* s_0.
2. $r(\Psi_{\bar{F}}) < 1$.

where $r(A)$ *denotes the spectral radius of matrix* A *and*

$$\Psi_{\bar{F}} \;\; = \;\; \left(\oplus_{s_0=1}^{S}\left(I_n - A_0^{(s_0)}E_0(\bar{\Omega}^{(s_1)})\right)^{-1}A_0^{(s_0)}\right)(P\otimes I_n)$$

and only if:

1. $\bar{\Omega}(s_0) = lim_{T_1\to\infty}\Omega_0(s_0)$ *exists for all* s_0.

2'. $\bar{\xi}(s_0) = lim_{T_1\to\infty}\xi_0(s_0)$ *exists for all* s_0.

[1] This corrigendum refers to equations in the published manuscript.

Proof: *see below.*

The remaining results of the paper are not affected by this error. Following the proof of corrected Proposition 2, this corrigendum provides an example where the sufficient conditions 1 and 2 in Proposition 2 are only sufficient. I want to note that such examples are not of practical interest in the forward guidance applications.

Proof of Corrected Proposition 2

First, we show that conditions 1 and 2' are necessary for ruling out the puzzle. The model does not exhibit the forward guidance puzzle if and only if: $\lim_{T_1\to\infty} x_0$ exists for all $s_0 \in \{1,\ldots,S\}$, given any $x_{-1} \in \mathbb{R}^n$, $u_0 \in \mathbb{R}^m$. We have:

$$\lim_{T_1\to\infty} x_0 \;=\; \lim_{T_1\to\infty}\left(\xi_0^{(s_0)} + \Omega_0^{(s_0)} x_{-1} + \Gamma_0^{(s_0)} u_0\right),$$

which clearly exists for any given x_{-1}, u_0, s_0 if and only if $\lim_{T_1\to\infty}\xi_0^{(s_0)} = \bar{\xi}^{(s_0)}$, $\lim_{T_1\to\infty}\Gamma_0^{(s_0)} = \bar{\Gamma}^{(s_0)}$, and $\lim_{T_1\to\infty}\Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$ exist for all s_0. Since (14) is independent of (15)-(16), $\lim_{T_1\to\infty}\Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$ exists independently of (15)-(16), and (15) reveals that $\bar{\Gamma}^{(s_0)}$ exists for all s_0 if $\bar{\Omega}^{(s_0)}$ exists for all s_0. Therefore, the forward guidance puzzle does not emerge only if $\lim_{T_1\to\infty}\xi_0^{(s_0)} = \bar{\xi}^{(s_0)}$ and $\lim_{T_1\to\infty}\Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$ exist for all s_0, such that $\lim_{T_1\to\infty} x_0 = \bar{\xi}^{(s_0)} + \bar{\Omega}^{(s_0)} x_{-1} + \bar{\Gamma}^{(s_0)} u_0$.

Second, we show that conditions 1 and 2 are sufficient for ruling out the puzzle. If condition 1 is satisfied then $\lim_{T_1\to\infty}\Gamma_0^{(s_0)} = \bar{\Gamma}^{(s_0)}$, and $\lim_{T_1\to\infty}\Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$ exist for all s_0, as argued above. Further if condition 1 holds, then as $T_1 \to \infty$, ξ_t evolves backwards in time (i.e. as $t \to -\infty$ and $T_1 \to \infty$) according to:[2]

$$\xi_t = \Psi_{\bar{F}}\xi_{t+1} + \Psi_{\bar{A}} D_0, \tag{1}$$

where $\xi_t = (\xi_t^{(1)'},\ldots,\xi_t^{(S)'})'$, $D_0 = (D_0^{(1)'},\ldots,D_0^{(S)'})'$,

$$\begin{aligned}\Psi_{\bar{F}} &= \left(\oplus_{s_0=1}^{S}\left(I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)})\right)^{-1} A_0^{(s_0)}\right)(P \otimes I_n),\\ \Psi_{\bar{A}} &= \left(\oplus_{s_0=1}^{S}\left(I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)})\right)^{-1}\right),\end{aligned}$$

and $\left(\oplus_{s_0=1}^{S} G(s_0)\right) = diag\left(G(1),\ldots,G(S)\right)$.

From (1), $\lim_{t\to-\infty}\xi_t = \lim_{T_1\to\infty}(\xi_0^{(1)'},\ldots,\xi_0^{(S)'})' = (\bar{\xi}^{(1)'},\ldots,\bar{\xi}^{(S)'})' = (I-\Psi_{\bar{F}})^{-1}\Psi_{\bar{A}} D_0$, if $r(\Psi_{\bar{F}}) < 1$. Therefore, the forward guidance puzzle does not emerge and $\lim_{T_1\to\infty} x_0 = \bar{\xi}^{(s_0)} + \bar{\Omega}^{(s_0)} x_{-1} + \bar{\Gamma}^{(s_0)} u_0$ if $\lim_{T_1\to\infty}\Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$ exists for all s_0 and $r(\Psi_{\bar{F}}) < 1$. ■

[2] Technically, our notation in the main text assumes $t \geq 0$, but with a slight abuse of notation we can redefine t to satisfy the claim that (1) governs the evolution of ξ_0 as $T_1 \to \infty$. It should also be recalled that $\left(I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})})\right)^{-1}$ is assumed to exist for all t, i and s_t in the underlying solution method.

Illustrative Example and Intuition

For example, consider the following forward guidance exercise, which is nested in the model class discussed in section 3 of the published manuscript and described by equations (11)-(12) therein:

$$i_t = E_t\pi_{t+1}, \tag{2}$$

$$i_t = \begin{cases} \phi_0\pi_t & \text{for } t = 0,\dots,T_1-1 \\ \phi_0\pi_t + \bar{i} & \text{for } t = T_1 \\ \phi_1\pi_t & \text{for } t > T_1 \end{cases},$$

where π is inflation, i is the nominal interest rate, (2) is the Fisher relation and $0 < \phi_0 < 1 < \phi_1$. The papers that comprise the New Keynesian forward guidance literature study announcements that entail an anticipated change in the path of interest rates ($\bar{i} \neq 0$).[3] In keeping with that literature, as well as the general definition of forward guidance puzzle given in section 3 of the published paper (see Definition 1), we say that a forward guidance puzzle does not emerge in our simple example if and only if $lim_{T_1\to\infty}\pi_0$ exists.

In this special application, the solution recursion (16) can be represented as:

$$\begin{aligned} \xi_{T_1} &= -\frac{\bar{i}}{\phi_0} + \frac{1}{\phi_0}E_{T_1}\xi_{T_1+1}, \\ \xi_t &= \frac{1}{\phi_0}E_t\xi_{t+1} \text{ for } t = 0,\dots,T_1-1, \\ \pi_t &= \xi_t \qquad\quad \text{for } t = 0,\dots,T_1, \end{aligned}$$

given $E_{T_1}\xi_{T_1+1} = 0$ (which is implied by $\phi_1 > 1$). The spectral radius term in condition 2 of Proposition 2 boils down to: $r(\Psi_{\bar{F}}) = \frac{1}{\phi_0} > 1$. Since there are no lagged endogenous variables in this model, condition 1 is satisfied but condition 2 of Proposition 2 is violated. If $\bar{i} \neq 0$, such that the forward guidance announcement entails an anticipated shock to the nominal interest rate as in standard forward guidance experiments, then the puzzle clearly emerges, which one can verify by iterating on the difference equation for ξ_t (i.e., $|\pi_0| \to \infty$ as $T_1 \to \infty$). However, in the special case with $\bar{i} = 0$, such that the central bank only announces a change in the inflation reaction coefficient (ϕ) without any corresponding change in the interest rate itself, the puzzle is absent. In that case, which is different from the forward guidance experiments considered in my paper and in the forward guidance puzzle literature (which assume a shock to the path of interest rates, e.g, $\bar{i} \neq 0$), both inflation and the interest rate remain in steady state ($\pi_t = i_t = 0$) for all $t \geq 0$.

The forward guidance puzzle is generally absent if $D_i^{(s_t)} = 0$ for all i and s_t in equation (12) and condition 1 of Proposition 2 is satisfied, but condition 2 fails. The

[3] Moreover, most papers in the literature assume an anticipated switch from passive to active monetary policy ($0 \leq \phi_0 < 1 < \phi_1$) around the time of the anticipated shock. See the published manuscript for a review of the literature. Also note that $\pi_t = -\bar{i}$ for all $t \leq T_1$ if $\phi_0 = 1$, but to my knowledge, this calibration is not practically relevant in forward guidance applications.

vector $D_i^{(s_t)}$ captures anticipated shocks to the level of endogenous or exogenous variables.[4] Canonical forward guidance announcements involve anticipated changes in the level and path of the interest rate, and hence they can be represented in the form (12), where $D_i^{(s_t)} \neq 0$ for some i and s_t. A forward guidance puzzle may emerge if condition 2 fails and $D_i^{(s_t)} \neq 0$ for some i and s_t, but only in exceptional cases.[5] In other words, failure to satisfy condition 2 indicates the existence of forward guidance announcements that give rise to the puzzle, even if the specific announcement under study does not generate counter-intuitive, explosive economic responses. The illustrative example makes this clear, as the puzzle is absent only if $\bar{i} = 0$, but not for alternative promises about the future path of interest rates ($\bar{i} \neq 0$). On the other hand, if condition 2 is satisfied (e.g., suppose $\phi_0 > 1$ in the simple example above), then the puzzle is absent under any value of the anticipated shock, $\bar{i}$. More generally, condition 2 of Proposition 2 ensures that predictions about the absence of the forward guidance puzzle are robust to small changes in the details of the announcement itself (i.e., the model is puzzle-free for *any* $D_i^{(s_t)}$ when conditions 1 and 2 hold). In other words, failure of conditions 1 and 2 is a warning signal that we can construct forward guidance announcements that have puzzling effects.

[4]We can augment the vector x_t to include exogenous variables as well.

[5]For an example of such a special case, consider the simple exercise above with $\phi_0 = 1$ such that $r(\Psi_{\bar{F}}) = 1$. If $\bar{i} = D_1 \neq 0$ there is no puzzle (while if we modify the problem such that $i_t = \phi_0 \pi_t + \bar{i}$ and $\bar{i} \neq 0$ for all $t = 0, \ldots, T_1$, as in some standard forward guidance experiments, then we do have a forward guidance puzzle). As mentioned in footnote 3, this case is not practically relevant in the forward guidance literature, and analogous cases are not encountered in my paper's analytical or numerical applications.